ORIGINAL ARTICLE

Robust Observer-based Control Design for Singular Systems with Delays in States

Ahmed Ech-charqy[1,*], Mohamed Ouahi[2], and El Houssaine Tissir[1]

[1] *Depaetment of Physics, Faculty of Sciences, University Sidi Mohammed Ben Abdellah, Fes, Morocco.*

[2] *LISA, ENSAF, BP 72, QI Ain Chkef Bensouda FES, Morocco.*

(**Received** 26 September 2017; **Accepted** 16 November 2017; **Published on line** 1 September 2018)

*Corresponding author: ahmed.echcharqy1@usmba.ac.ma

DOI: 10.5875/ausmt.v8i3.1611

Abstract: In this paper, the robust observer-based control problem for uncertain singular time delay systems is investigated. The approach is based on using Finsler's lemma. First, delay-dependent linear matrix inequality (LMI) conditions are obtained which, guarantee that the nominal unforced system is regular, impulse free, and stable. Then, with the help of obtained admissibility criterion, an observer-based controller is given by solving a set of LMIs. Finally, some simulations are provided to demonstrate the effectiveness of the obtained results.

Keywords: admissibility; singular time delay systems; observer-based control; linear matrix inequality

Introduction

Time delays constitute an inherent feature of several dynamic systems; they are regarded as an important source of instability and performance degradation in a great number of important engineering problems involving material, information or energy transportation. During the past three decades, considerable attention has been devoted to the analysis and synthesis of these time delay systems via linear matrix inequality (LMI) approach, for example, the Finsler's lemma was combined with the Lyapunov-Krasovskii functional and linear matrix inequality (LMI) techniques [10], the reciprocally convex approach [11], the novel integral equalities method [12], and the augmented Lyapunov functional with some triple integral terms method [13]. Very recently, the Wirtinger based integral inequality [14] was presented to achieve less conservative results.

On the other hand, the singular systems, also referred to as descriptor systems, generalized state-space systems, differential-algebraic systems or semi-state systems. Over the past decades, it has extensive applications in many areas such as economic systems, chemical process, circuit systems, electric systems, power systems, robotic systems, networked control systems, space navigation systems, biological systems and other areas [1,2]. It should be pointed out that the stability problem of singular systems is much more complicated than state-space systems, because it requires considering not only stability, but also it required regularity and impulse-free conditions (for continuous systems).

During the recent years, much attention has been devoted to the study of singular systems with time-delay. As time delay is often encountered in various practical systems, and the fact that the existence of time delay often causes performance degradation or instability in dynamical systems. The methods may be classified into two categories: delay-independent cases [6] and delay-dependent cases [7]. Generally speaking, delay-independent cases are likely to be conservative, especially when the delay is comparatively small. Therefore, more attention has been paid to the study of delay-dependent stability of singular time-delay system and several results are obtained [8,9].

Many approaches have been developed in the literature to reduce the conservatism of the delay-dependent stability of singular time-delay system. For example, in [23], the model transformation method is proposed. The delay partitioning approach is developed in [26,27]. By Considering the appropriate

Lyapunov-Krasovskii functional based on Jensen Inequality, delay dependent conditions are given in [3,9,25]. Very recently, the Wirting-based integral inequality to achieve less conservative results, however, stability analysis of singular time-delay system based on the Wirtinger integral inequality approach has rarely been studied. Note that all the above references assume that the system's state is measurable. Hence, if the state is note fully measurable, these works can't be applied.

It has been recognized that in most practical situations, state variables are generally not easily available through output measurement. In this case, the problem of state feedback controller design for time-delay systems is infeasible. Therefore, the design of a controller that does not require complete access to the state vector is preferable. That is why many observer design problems, which are concerned in using the available information on inputs and outputs to reconstruct the unmeasured states of the studied system, have been widely investigated for many practical applications in [15-19], whereas only a few works have been carried out on singular systems [20-22] this motivates our work.

The objective of this paper is to design an observer-based controller such that the uncertain singular time-delay system is robustly stable. First, a delay-dependent LMI condition that guarantees that the nominal unforced system is regular, impulse-free, and stable. A lemma will play a significant role in the derivation of a less conservative delay-dependent result. Then, the problem of robust output feedback stabilization is examined using an observer-based control law. Finally, some numerical examples are provided to demonstrate the efficiency of the proposed results.

Notation: X^T denotes the transpose of X. X^+ denotes the Moore-Penrose pseudo-inverse matrix of X. Symmetric elements in the matrix are denoted by $*$. $Sym(X)$ denotes $X+X^T$. $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote respectively the n-dimensional Euclidean space and the set of all $n\times m$ real matrices.

Ahmed Ech-charqy received her Master on signals, systems and Computing from the Faculty of Sciences of the University Sidi Mohammed Ben Abdellah, Morocco, in 2015. Her research interests include observer based control of singular systems with time delay.
Email: ahmed.echcharqy1@usmba.ac.ma

Mohamed Ouahi received the sciences degree in automatic systems and image in 2008 and Ph.D degrees in computer engineering and automatic in 2011 from National School of Engineers of Limoge France, He becomes a researcher in National School of Applied Sciences, Fes, Morocco, since 2013. His current research interests include nonlinear control theory, state observer, the automotive supervision and singular systems.
Email: mohamed.ouahi@usmba.ac.ma

El Houssaine Tissir received his Diploma of higher studies and Doctorate of state degree from the University Sidi Mohammed Ben Abellah, at Fes, Morocco in 1992 and 1997, respectively. He has been with the Department of Physics, in the same university, Dhar El Mehraz at Fes, where he is currently a Full Professor. His current research interests include robust control, time delay systems, systems with saturating actuators, H_∞ control, singular systems and neutral systems.
Email: elhoussaine.tissir@usmba.ac.ma

Problem description and preliminaries

Consider the following uncertain singular time delay systems

$$\begin{cases} E\dot{x}(t)=(A+\Delta A(t))x(t)+(A_h+\Delta A_h(t))x(t-h) \\ \qquad +(B+\Delta B(t))u(t), \\ y(t)=Cx(t), \\ x(t)=\phi(t),\ t\in[-\bar{h},0]. \end{cases} \tag{1}$$

Where $x(t)\in\mathbb{R}^n$ is the state vector, $u(t)\in\mathbb{R}^m$ is the control input and $y(t)\in\mathbb{R}^p$ represents the measured output vector, h is a constant delay satisfying $0<h\le\bar{h}$ and $\phi(t)$ is the initial condition of the system. The matrix $E\in\mathbb{R}^{n\times n}$, let $rank(E)=r\le n$, i.e matrix E may be singular. Matrices A, A_h, B and C are known real constant matrices with appropriate dimensions. $\Delta A(t)$, $\Delta A_h(t)$ and $\Delta B(t)$ are unknown matrices representing norm-bounded parametric uncertainties and are assumed to be of the form

$$[\Delta A(t), \Delta A_h(t), \Delta B(t)] = HF(t)[E_1, E_2, E_3] \tag{2}$$

where H, E_1, E_2, E_3 are known real constant matrices with appropriate dimensions and $F(t)$ is are unknown real and possibly time-varying matrices satisfying

$$F(t)^T F(t)\le \mathrm{I} \tag{3}$$

The following lemmas are very useful for our development in this paper.

Lemma 1: [29]. Let $Q=Q^T, H, E$ and $F(t)$ satisfying $F(t)^T F(t)\le \mathrm{I}$ appropriately dimensioned matrices, the following inequality $Q+HF(t)E+E^TF(t)^TH^T<0$ is true, if and only if there exist a scalar $\varepsilon>0$ such that

$$Q+\varepsilon HH^T+\varepsilon^{-1}E^TE<0 \tag{4}$$

Lemma 2: [30]. Consider a given matrix $\chi\in\mathbb{R}^n$, a symmetric matrix $Q\in\mathbb{R}^{n\times n}$ and a matrix $\mathrm{B}\in\mathbb{R}^{m\times n}$, such that $rank(B)<n$. The following statements are equivalent:

1) $\chi^T Q\chi<0, \forall\chi$ such that $B\chi=0, \chi\ne 0$
2) $B^{\perp T}QB^{\perp}<0$
3) $\exists\mu\in\mathbb{R}: Q-\mu B^TB<0$
4) $\exists F\in\mathbb{R}^{n\times m}: Q+FB+B^TF^T<0$

where $B^{\perp}$ denotes a basis for the null-space of B.

Lemma 3: [14]. Consider a given matrix $T>0$; then, for all continous function ω in $[a,b]\to\mathbb{R}^n$, the following inequality holds:

$$(b-a)\int_a^b \omega(s)^T T\omega(s)ds \geq \left(\int_a^b \omega(s)ds\right)^T T\left(\int_a^b \omega(s)ds\right)+3\Omega^T T\Omega$$

where

$$\Omega=\int_a^b \omega(s)ds-\frac{2}{b-a}\int_a^b\int_a^s \omega(r)dr\,ds \quad .$$

Definition 1: [1]. 1) The pair (E,A) is said to be regular if $\det(sE-A)\neq 0$. 2) The pair (E,A) is said to be impulse free if $\deg(\det(sE-A))=rank(E)$.

Definition 2: [1]. 2) The singular time-delay system

$$\begin{cases} E\,\dot{x}(t)=A\,x(t)+A_h\,x(t-h) \\ x(t)=\phi(t),\, t\in\left[-\bar{h},0\right] \end{cases} \tag{5}$$

is said to be regular and impulse free, if the pair (E,A) is regular and impulse free.

2) System (6) is said to be asymptotically stable, if for any, there exists a scalar $\delta(\varepsilon)>0$, such that for any compatible initial condition $\phi(t)$ with, $\sup_{-h<t<0}\|\phi(t)\|<\delta(t)$ The solution *x(t)* of (5) satisfies $\|x(t)\|<\varepsilon$ for $t>0$ and $\lim_{t\to 0} x(t)=0$.

3) System (5) is said to be admissible, if it is regular, impulse free, and asymptotically stable.

Mains results

Delay-dependent stability analysis for singular system

In this section, we concentrate our attention on the problem of stability analysis for nominal system (1).

Theorem 1: For a given scalar $h>0$, system described by (5) is admissible if there exists symmetric positive definite matrices $R_1, Q_{11}, Q_{22}, P_{11}, P_{22}$ and appropriately dimensioned matrices $Q_{12}, P_{12}, F_0, F_1, F_2$ such that the following LMIs are feasible.

$$\Upsilon=\begin{bmatrix} \Upsilon_{11} & \Upsilon_{12} & \Upsilon_{13} & \Upsilon_{14} & \Upsilon_{15} \\ * & \Upsilon_{22} & \Upsilon_{23} & \Upsilon_{24} & \Upsilon_{25} \\ * & * & \Upsilon_{33} & \Upsilon_{34} & \Upsilon_{35} \\ * & * & * & \Upsilon_{44} & \Upsilon_{45} \\ * & * & * & * & \Upsilon_{55} \end{bmatrix}<0 \tag{6}$$

$$\begin{bmatrix} Q_{11} & Q_{12} \\ * & Q_{22} \end{bmatrix}>0 \tag{7}$$

$$\begin{bmatrix} P_{11} & P_{12} \\ * & P_{22} \end{bmatrix}>0 \tag{8}$$

where

$$\Upsilon_{11}=R_1+hQ_{11}-\frac{4}{h}E^TQ_{22}E+E^TP_{12}+P_{12}^TE+F_0A+A^TF_0^T$$

$$\Upsilon_{12}=-\frac{2}{h}E^TQ_{22}E-E^TP_{12}+F_0A_h+A^TF_1^T$$

$$\Upsilon_{13}=hQ_{12}+SR^T+E^TP_{11}-F_0+A^TF_2^T$$

$$\Upsilon_{14}=\frac{6}{h^2}E^TQ_{22}E-\frac{4}{h}E^TQ_{12}^T+P_{22}$$

$$\Upsilon_{15}=\frac{6}{h^2}E^TQ_{12}^T$$

$$\Upsilon_{22}=-R_1-\frac{4}{h}E^TQ_{22}E+F_1A_h+A_h^TF_1^T$$

$$\Upsilon_{23}=-F_1+A_h^TF_2^T$$

$$\Upsilon_{24}=\frac{6}{h^2}E^TQ_{22}E-\frac{2}{h}E^TQ_{12}^T-P_{22}$$

$$\Upsilon_{25}=\frac{6}{h^2}E^TQ_{12}^T$$

$$\Upsilon_{33}=hQ_{22}-F_2-F_2^T$$

$$\Upsilon_{34}=P_{12}$$

$$\Upsilon_{35}=0$$

$$\Upsilon_{44}=\frac{6}{h^2}\left(E^TQ_{12}^T+Q_{12}E\right)-\frac{4}{h}Q_{11}-\frac{12}{h^3}E^TQ_{22}E$$

$$\Upsilon_{45}=\frac{6}{h^2}Q_{11}-\frac{12}{h^3}E^TQ_{12}^T$$

$$\Upsilon_{55}=-\frac{12}{h^3}Q_{11}.$$

Proof: The proof is divided into two parts. First, we deal with the regularity and impulse-free properties. Second, we treat the stability property. First, we show that system (5) is regular and impulse free. We choose two nonsingular matrices M and N such that

$$\bar{E}=MEN=\begin{bmatrix} \mathrm{I}_r & 0 \\ 0 & 0 \end{bmatrix}$$

$$\bar{A}=MAN=\begin{bmatrix} \bar{A}_{11} & \bar{A}_{12} \\ \bar{A}_{21} & \bar{A}_{22} \end{bmatrix}$$

$$S=N^TS=\begin{bmatrix} \bar{S}_{11} \\ \bar{S}_{21} \end{bmatrix}.$$

Noting that $E^TR=0$ and $R\in\mathbb{R}^{n\times(n-r)}$, we can obtain

$$M^{-T}R=\begin{bmatrix} 0 \\ \bar{R}_{22} \end{bmatrix}.$$

From the properties of negative definite matrices, it can be easily seen that:

$$\begin{bmatrix} \Upsilon_{11} & \Upsilon_{13} \\ * & \Upsilon_{33} \end{bmatrix} < 0 \tag{9}$$

Let $J = \begin{bmatrix} \mathrm{I} & A^T \end{bmatrix}$, Pre-multiplying and post-multiplying (9) by J and J^T, respectively, we get

$$\begin{aligned} &sym\left(E^TP_{12} + E^TP_{11}A + hQ_{12}A + SR^TA\right) + R_1 + hQ_{11} \\ &\quad - \frac{4}{h}E^TQ_{22} + A^TQ_{22}A < 0 \end{aligned} \tag{10}$$

Pre-multiplying and post-multiplying (10) N^T and N, we have $sym\left(\bar{S}_{21}\bar{R}_{22}^T\bar{A}_{22}\right) < 0$, which means $\bar{A}_{22}$ is nonsingular. Then, by Definition 1, the pair (E, A) is regular and impulse-free. Therefore, from Definition 2, the system (5) is regular and impulse-free.

Next, we show the stability of system (5). Choose the following Lyapunov-Krasovskii functional

$$V(x_t) = V_1(x_t) + V_2(x_t) + V_3(x_t) \tag{11}$$

$$V_1(x_t) = \int_{t-h}^{t} x^T(s)R_1x(s)ds \tag{12}$$

$$V_2(x_t) = \begin{bmatrix} Ex(t) \\ \int_{t-h}^{t} x(s)ds \end{bmatrix}^T \begin{bmatrix} P_{11} & P_{12} \\ * & P_{22} \end{bmatrix} \begin{bmatrix} Ex(t) \\ \int_{t-h}^{t} x(s)ds \end{bmatrix} \tag{13}$$

$$V_3(x_t) = \int_{-h}^{0}\int_{t+\theta}^{t} \eta(s)^TQ\eta(s)ds\,d\theta \tag{14}$$

Where $x_t = x(t+\alpha)$ for $\alpha \in [-h, 0]$ and $\eta(t) = \begin{bmatrix} x(t)^T & (E\dot{x}(t))^T \end{bmatrix}^T$.

Along the solution of system (5); the time derivative of $V(x_t)$ is given by:

$$\dot{V}_1(x_t) = x^T(t)R_1x(t) - x^T(t-h)R_1x(t-h) \tag{15}$$

$$\dot{V}_2(x_t) = 2\begin{bmatrix} Ex(t) \\ \int_{t-h}^{t} x(s)ds \end{bmatrix}^T \begin{bmatrix} P_{11} & P_{12} \\ * & P_{22} \end{bmatrix} \begin{bmatrix} E\dot{x}(t) \\ x(t) - x(t-h) \end{bmatrix} \tag{16}$$

$$\dot{V}_3(x_t) = h\eta(t)^TQ\eta(t) - \int_{t-h}^{t} \eta(s)^TQ\eta(s)ds \tag{17}$$

By applying Lemma 3 we can obtain:

$$\dot{V}_3(x_t) \le h\eta(t)^TQ\eta(t) - \frac{1}{h}\left(\int_{t-h}^{t} \eta(s)ds\right)^T Q\left(\int_{t-h}^{t} \eta(s)ds\right) - \frac{3}{h}\Xi^TQ\Xi \tag{18}$$

Where

$$\Xi = \int_{t-h}^{t} \eta(s)\,ds - \frac{2}{h}\int_{t-h}^{t}\int_{t-h}^{s} \eta(u)du\,ds.$$

Noting that $E^TR = 0$, we can deduce that

$$2\dot{x}(t)E^TRS^Tx(t) = 0 \tag{19}$$

Combining (15), (16), (18) and (19) yields

$$\dot{V}(x_t) \le \xi^T(t)\Phi\xi(t) \tag{20}$$

Where

$$\xi^T(t) = \left(x^T(t) \quad x^T(t-h) \quad (E\dot{x}(t))^T \quad \left(\int_{t-h}^{t} x(s)ds\right)^T \quad \left(\int_{t-h}^{t}\int_{t-h}^{s} x(u)du\,ds\right)^T \right) \tag{21}$$

$$\Phi = \begin{bmatrix} \Phi_{11} & \Phi_{12} & \Phi_{13} & \Phi_{14} & \Phi_{15} \\ * & \Phi_{22} & 0 & \Phi_{24} & \Phi_{25} \\ * & * & \Phi_{33} & \Phi_{34} & 0 \\ * & * & * & \Phi_{44} & \Phi_{45} \\ * & * & * & * & \Phi_{55} \end{bmatrix} \tag{22}$$

and

$$\Phi_{11} = R_1 + hQ_{11} - \frac{4}{h}E^TQ_{22}E + E^TP_{12} + P_{12}^TE$$

$$\Phi_{12} = -\frac{2}{h}E^TQ_{22}E - E^TP_{12}$$

$$\Phi_{13} = hQ_{12} + SR^T + E^TP_{11}$$

$$\Phi_{14} = \frac{6}{h^2}E^TQ_{22}E - \frac{4}{h}E^TQ_{12}^T + P_{22}$$

$$\Phi_{15} = \frac{6}{h^2}E^TQ_{12}^T$$

$$\Phi_{22} = -R_1 - \frac{4}{h}E^TQ_{22}E$$

$$\Phi_{24} = \frac{6}{h^2}E^TQ_{22}E - \frac{2}{h}E^TQ_{12}^T - P_{22}$$

$$\Phi_{25} = \frac{6}{h^2}E^TQ_{12}^T$$

$$\Phi_{33} = hQ_{22}$$

$$\Phi_{34} = P_{12}$$

$$\Phi_{44} = \frac{6}{h^2}\left(E^TQ_{12}^T + Q_{12}E\right) - \frac{4}{h}Q_{11} - \frac{12}{h^3}E^TQ_{22}E$$

$$\Phi_{45} = \frac{6}{h^2}Q_{11} - \frac{12}{h^3}E^TQ_{12}^T$$

$$\Phi_{55} = -\frac{12}{h^3}Q_{11}.$$

Now, let

$$\bar{B} = \begin{bmatrix} A & A_h & -\mathrm{I} & 0 & 0 \end{bmatrix}$$

$$F = \begin{bmatrix} F_0^T & F_1^T & F_2^T & 0 & 0 \end{bmatrix}^T$$

Then we can verify that $\bar{B}\xi = 0$. The matrix Υ in (6) can be written as:

$$\Upsilon = \Phi + F\bar{B} + \bar{B}^T F^T < 0.$$

Applying Lemma 2 we have $\xi^T(t)\Phi\xi(t) < 0$ which implies that $\dot{V}(x_t) < 0$. Thus, the system (5) is asymptotically stable. The proof is completed.

Remark 1: We addressed the problem of the stability analysis of singular systems with delay in state, by using Finsler's lemma. To the best of our knowledge, Finsler's lemma has been used in [10] and [28] where it has been shown via extensive numerical computations that it gives less conservative results than the other approaches. This approach has also the advantage, that it leads to conditions that to not include products of the Lyapunov matrices with neither the instantaneous state matrix A nor the retarded state matrix A_h.

Remark 2: Theorem 1 gives a delay-dependent sufficient condition, which guarantees that system (5) is regular, impulse-free and stable. From (6), (7) and (8), it is seen that theorem 1 doesn't rely on a condition of type $E^T P = P^T E \geq 0$ which is difficult to implement on LMI Toolbox of Matlab, as in the case of most of literature results [3,7,9,23-27]. Consequently, the complexity of implementation of our results is reduced.

Observer-based control design

In the following, the problem that we are dealing with is observer-based controller design for singular system (1). More specially, we shall give sufficient condition for the existence of controller in terms of LMIs and present the corresponding observer-based controller design method. To achieve the aforementioned objective, we use the following observer-based controller:

$$\begin{cases} E\dot{\hat{x}}(t) = A\hat{x}(t) + A_h\hat{x}(t-h) + Bu(t) + L\left(y(t) - \hat{y}(t)\right), \\ \hat{y}(t) = C\hat{x}(t), \\ u(t) = K\hat{x}(t), \\ x(t) = \psi(t),\ t \in [-\bar{h}, 0]. \end{cases} \tag{23}$$

Where $\hat{x}(t)$ is the estimated state, K and L are the controller gain and the observer gain to be designed, respectively. Define the estimation error as $e(t) = x(t) - \hat{x}(t)$, and $\tilde{x}(t) = \begin{bmatrix} \hat{x}^T(t) & e^T(t) \end{bmatrix}^T$. Combining (1) with (23), the augmented closed-loop system is written as

$$\begin{cases} \tilde{E}\dot{\tilde{x}}(t) = \tilde{A}(t)\tilde{x}(t) + \tilde{A}_h\tilde{x}(t-h) \\ \tilde{x}(t) = \left[\psi(t), \left(\phi(t) - \psi(t)\right)\right], t \in [-\bar{h}, 0] \end{cases} \tag{24}$$

with

$$\tilde{A}(t) = \tilde{A} + \tilde{H}\tilde{F}(t)\tilde{E}_A,\ \tilde{A}_h(t) = \tilde{A}_h + \tilde{H}_h F(t)\tilde{E}_h, \tilde{F}(t) = diag\left(F(t), F(t)\right),$$

$$\tilde{E} = \begin{bmatrix} E & 0 \\ 0 & E \end{bmatrix}, \tilde{A} = \begin{bmatrix} A + BK & LC \\ 0 & A - LC \end{bmatrix}, \tilde{A}_h = \begin{bmatrix} A_h & 0 \\ 0 & A_h \end{bmatrix},$$

$$\tilde{H} = \begin{bmatrix} 0 & 0 \\ H & H \end{bmatrix}, \tilde{E}_A = \begin{bmatrix} E_1 & E_1 \\ E_3K & 0 \end{bmatrix}, \tilde{H}_h = \begin{bmatrix} 0 \\ H \end{bmatrix}, \tilde{E}_h = \begin{bmatrix} E_2 & E_2 \end{bmatrix}.$$

Remark 3: In the case when $E = \mathrm{I}$, that is to say, the singular systems are reduced into linear systems. The problem of the observer-based controller design for linear systems is considered in [15-19]. However, the results of these papers can't be applied to singular systems. This makes our results have broader applicability in practice.

Theorem 2: For a given scalar $h > 0, \varepsilon_1 > 0, \varepsilon_2 > 0$ and scalar tuning parameters $\mu_i \neq 0, i = 1,2$, the system described by (24) is admissible if there exists symmetric positive definite matrices $\tilde{R}_1, \tilde{Q}_{11}, \tilde{Q}_{22}, \tilde{P}_{11}, \tilde{P}_{22}$ and appropriately dimensioned matrices $\tilde{Q}_{12}, \tilde{P}_{12}, \tilde{X}$ such that the following conditions are feasible.

$$\tilde{\Upsilon} = \begin{bmatrix} \tilde{\Upsilon}_{11} & \tilde{\Upsilon}_{12} & \tilde{\Upsilon}_{13} & \tilde{\Upsilon}_{14} & \tilde{\Upsilon}_{15} & \tilde{\Upsilon}_{16} & \tilde{\Upsilon}_{17} \\ * & \tilde{\Upsilon}_{22} & \tilde{\Upsilon}_{23} & \tilde{\Upsilon}_{24} & \tilde{\Upsilon}_{25} & \mu_1\tilde{\Upsilon}_{16} & \mu_1\tilde{\Upsilon}_{17} \\ * & * & \tilde{\Upsilon}_{33} & \tilde{\Upsilon}_{34} & \tilde{\Upsilon}_{35} & \mu_2\tilde{\Upsilon}_{16} & \mu_2\tilde{\Upsilon}_{17} \\ * & * & * & \tilde{\Upsilon}_{44} & \tilde{\Upsilon}_{45} & 0 & 0 \\ * & * & * & * & \tilde{\Upsilon}_{55} & 0 & 0 \\ * & * & * & * & * & -\varepsilon_1\mathrm{I} & 0 \\ * & * & * & * & * & * & -\varepsilon_2\mathrm{I} \end{bmatrix} < 0 \tag{25}$$

$$\begin{bmatrix} \tilde{Q}_{11} & \tilde{Q}_{12} \\ * & \tilde{Q}_{22} \end{bmatrix} > 0 \tag{26}$$

$$\begin{bmatrix} \tilde{P}_{11} & \tilde{P}_{12} \\ * & \tilde{P}_{22} \end{bmatrix} > 0 \tag{27}$$

where

$$\tilde{\Upsilon}_{11} = \tilde{R}_1 + h\tilde{Q}_{11} - \frac{4}{h}\tilde{E}\tilde{Q}_{22}\tilde{E}^T + \tilde{E}\tilde{P}_{12} + \tilde{P}_{12}^T\tilde{E}^T + \tilde{A}\tilde{X} + \tilde{X}^T\tilde{A}^T + \varepsilon_1\tilde{H}\tilde{H}^T$$

$$\tilde{\Upsilon}_{12} = -\frac{2}{h}\tilde{E}\tilde{Q}_{22}\tilde{E}^T - \tilde{E}\tilde{P}_{12} + \tilde{X}^T\tilde{A}_h^T + \mu_1\tilde{A}\tilde{X}$$
$$\tilde{\Upsilon}_{13} = h\tilde{Q}_{12} + \tilde{S}\tilde{R}^T + \tilde{E}\tilde{P}_{11} - \tilde{X}^T + \mu_2\tilde{A}\tilde{X}$$
$$\tilde{\Upsilon}_{14} = \frac{6}{h^2}\tilde{E}\tilde{Q}_{22}\tilde{E}^T - \frac{4}{h}\tilde{E}\tilde{Q}_{12}^T + \tilde{P}_{22}$$
$$\tilde{\Upsilon}_{15} = \frac{6}{h^2}\tilde{E}\tilde{Q}_{12}^T$$
$$\tilde{\Upsilon}_{16} = \tilde{X}^T\tilde{E}_A^T$$
$$\tilde{\Upsilon}_{17} = \tilde{X}^T\tilde{E}_h^T$$
$$\tilde{\Upsilon}_{22} = -\tilde{R}_1 - \frac{4}{h}\tilde{E}\tilde{Q}_{22}\tilde{E}^T + \mu_1\tilde{X}^T\tilde{A}_h^T + \mu_1\tilde{A}_h\tilde{X} + \varepsilon_2\tilde{H}_h\tilde{H}_h^T$$
$$\tilde{\Upsilon}_{23} = -\mu_1\tilde{X}^T + \mu_2\tilde{A}_h\tilde{X}$$
$$\tilde{\Upsilon}_{24} = \frac{6}{h^2}\tilde{E}\tilde{Q}_{22}\tilde{E}^T - \frac{2}{h}\tilde{E}\tilde{Q}_{12}^T - \tilde{P}_{22}$$
$$\tilde{\Upsilon}_{25} = \frac{6}{h^2}\tilde{E}\tilde{Q}_{12}^T$$
$$\tilde{\Upsilon}_{33} = h\tilde{Q}_{22} - \mu_2\tilde{X}^T - \mu_2\tilde{X}$$
$$\tilde{\Upsilon}_{34} = \tilde{P}_{12}$$
$$\tilde{\Upsilon}_{35} = 0$$
$$\tilde{\Upsilon}_{44} = \frac{6}{h^2}\left(\tilde{E}\tilde{Q}_{12}^T + \tilde{Q}_{12}\tilde{E}^T\right) - \frac{4}{h}\tilde{Q}_{11} - \frac{12}{h^3}\tilde{E}\tilde{Q}_{22}\tilde{E}^T$$
$$\tilde{\Upsilon}_{45} = \frac{6}{h^2}\tilde{Q}_{11} - \frac{12}{h^3}\tilde{E}\tilde{Q}_{12}^T$$
$$\tilde{\Upsilon}_{55} = -\frac{12}{h^3}\tilde{Q}_{11}.$$

and $\tilde{R}\in\mathbb{R}^{n\times(n-r)}$ is any matrix with a full column rank that satisfies $\tilde{E}^T\tilde{R}=0$, $rank(\tilde{R})=n-r$.

Proof: Under the conditions of the theorem, it follows from $h\tilde{Q}_{22} - \mu_2\tilde{X}^T - \mu_2\tilde{X} < 0$ that $\tilde{X}$ is nonsingular.
Now, consider the following singular delay system:

$$\tilde{E}^T\dot{\zeta}(t) = \tilde{A}^T\zeta(t) + \tilde{A}_h^T\zeta(t-h) \tag{28}$$

Note that if $\det\left(s\tilde{E}-\tilde{A}\right) = \det\left(s\tilde{E}^T - \tilde{A}^T\right)$, then the pair $(\tilde{E},\tilde{A})$ is regular, impulse-free, and stable if and only if the pair $(\tilde{E}^T,\tilde{A}^T)$ is regular, impulse-free, and stable. Moreover, because $\det\left(s\tilde{E}-\tilde{A}-e^{-hs}\tilde{A}_h\right)=0$ and $\det\left(s\tilde{E}^T-\tilde{A}^T-e^{-hs}\tilde{A}_h^T\right)=0$ have the same solution. As long as the regularity, free of impulse and stability are concerned, we can consider system (28) instead of (24) without uncertainties. Then, applying theorem 1 to the system (28) and setting $F_0^T = X, F_1^T = \mu_1 X \text{ and } F_2^T = \mu_2 X$, the following inequality holds:

$$\tilde{\Upsilon} = \begin{bmatrix} \tilde{\Upsilon}_{11} & \tilde{\Upsilon}_{12} & \tilde{\Upsilon}_{13} & \tilde{\Upsilon}_{14} & \Upsilon_{15} \\ * & \tilde{\Upsilon}_{22} & \tilde{\Upsilon}_{23} & \tilde{\Upsilon}_{24} & \tilde{\Upsilon}_{25} \\ * & * & \tilde{\Upsilon}_{33} & \tilde{\Upsilon}_{34} & \tilde{\Upsilon}_{35} \\ * & * & * & \tilde{\Upsilon}_{44} & \tilde{\Upsilon}_{45} \\ * & * & * & * & \tilde{\Upsilon}_{55} \end{bmatrix} < 0 \tag{29}$$

For the uncertain singular system (24), by replacing in (29) $\tilde{A}$ and $\tilde{A}_h$ with $\tilde{A}+\tilde{H}\tilde{F}(t)\tilde{E}_A$ and $\tilde{A}_h+\tilde{H}_hF(t)\tilde{E}_h$, respectively, yields the following inequality

$$\tilde{\Upsilon} + sym\left(\tilde{M}_1\tilde{F}(t)\tilde{N}_1 + \tilde{M}_2F(t)\tilde{N}_2\right) < 0 \tag{30}$$

where
$$\tilde{M}_1^T = \begin{bmatrix}\tilde{H}^T & 0 & 0 & 0 & 0\end{bmatrix},$$
$$\tilde{N}_1 = \begin{bmatrix}\tilde{E}_A\tilde{X} & \mu_1\tilde{E}_A\tilde{X} & \mu_2\tilde{E}_A\tilde{X} & 0 & 0\end{bmatrix},$$
$$\tilde{M}_2^T = \begin{bmatrix}0 & \tilde{H}_h^T & 0 & 0 & 0\end{bmatrix},$$
$$\tilde{N}_2 = \begin{bmatrix}\tilde{E}_h\tilde{X} & \mu_1\tilde{E}_h\tilde{X} & \mu_2\tilde{E}_h\tilde{X} & 0 & 0\end{bmatrix},$$

By Lemma 1, it follows that there exist scalars $\varepsilon_1 > 0$ and $\varepsilon_2 > 0$ such that

$$\tilde{\Upsilon} + \varepsilon_1\tilde{M}_1\tilde{M}_1^T + \varepsilon_1^{-1}\tilde{N}_1^T\tilde{N}_1 + \varepsilon_2\tilde{M}_2\tilde{M}_2^T + \varepsilon_2^{-1}\tilde{N}_2^T\tilde{N}_2 < 0 \tag{31}$$

Invoking the Schur complement, condition (25) holds.

Remark 4: Theorem 2 presents sufficient conditions on admissibility for system (1) via observer-based controller. According to the expressions of closed-loop system matrices, it can be seen that the conditions in the aforementioned Theorem 2 are nonlinear matrix inequalities (nLMIs) with respect to the parameter matrices K and L, since some terms of these conditions are appearing in (25) in nonlinear fashion. In order to facilitate solving theorem 2, some matrix transformations are needed to transform then nLMIs (25) to LMIs.

Theorem 3: For prescribed scalars $h>0, \varepsilon_1>0$, $\varepsilon_2>0$ and scalar tuning parameters $\mu_i \neq 0, i=1,2$, the considered observer-based controlling problem of the uncertain singular time-delay system (24) is solvable, if there exists symmetric positive definite matrices $\tilde{R}_1, \tilde{Q}_{11}, \tilde{Q}_{22}, \tilde{P}_{11}, \tilde{P}_{22}$ and appropriately dimensioned matrices $X_1\in\mathbb{R}^{n\times n}$, $\hat{X}_{11}\in\mathbb{R}^{q\times q}$, $\hat{X}_{21}\in\mathbb{R}^{(n-q)\times n}$, $\hat{X}_{22}\in\mathbb{R}^{(n-q)\times(n-q)}$, $\tilde{Q}_{12}, \tilde{P}_{12}$ such that the following LMIs are feasible.

$$\tilde{\Psi}=\begin{bmatrix}\tilde{\Psi}_{11} & \tilde{\Psi}_{12} & \tilde{\Psi}_{13} & \tilde{\Psi}_{14} & \tilde{\Psi}_{15} & \tilde{\Psi}_{16} & \tilde{\Psi}_{17}\\ * & \tilde{\Psi}_{22} & \tilde{\Psi}_{23} & \tilde{\Psi}_{24} & \tilde{\Psi}_{25} & \mu_1\tilde{\Psi}_{16} & \mu_1\tilde{\Psi}_{17}\\ * & * & \tilde{\Psi}_{33} & \tilde{\Psi}_{34} & \tilde{\Psi}_{35} & \mu_2\tilde{\Psi}_{16} & \mu_2\tilde{\Psi}_{17}\\ * & * & * & \tilde{\Psi}_{44} & \tilde{\Psi}_{45} & 0 & 0\\ * & * & * & * & \tilde{\Psi}_{55} & 0 & 0\\ * & * & * & * & * & -\varepsilon_1 I & 0\\ * & * & * & * & * & * & -\varepsilon_2 I\end{bmatrix}<0 \tag{32}$$

$$\begin{bmatrix}\tilde{Q}_{11} & \tilde{Q}_{12}\\ * & \tilde{Q}_{22}\end{bmatrix}>0 \tag{33}$$

$$\begin{bmatrix}\tilde{P}_{11} & \tilde{P}_{12}\\ * & \tilde{P}_{22}\end{bmatrix}>0 \tag{34}$$

where

$$\tilde{\Psi}_{11}=\tilde{R}_1+h\tilde{Q}_{11}-\frac{4}{h}\tilde{E}\tilde{Q}_{22}\tilde{E}^T+\tilde{E}\tilde{P}_{12}+\tilde{P}_{12}^T\tilde{E}^T+\tilde{A}\tilde{X}+\tilde{X}^T\tilde{A}^T+\varepsilon_1\tilde{H}\tilde{H}^T$$

$$\tilde{\Psi}_{12}=-\frac{2}{h}\tilde{E}\tilde{Q}_{22}\tilde{E}^T-\tilde{E}\tilde{P}_{12}+\tilde{X}^T\tilde{A}_h^T+\mu_1\tilde{A}\tilde{X}$$

$$\tilde{\Psi}_{13}=h\tilde{Q}_{12}+\tilde{S}\tilde{R}^T+\tilde{E}\tilde{P}_{11}-\tilde{X}^T+\mu_2\tilde{A}\tilde{X}$$

$$\tilde{\Psi}_{14}=\frac{6}{h^2}\tilde{E}\tilde{Q}_{22}\tilde{E}^T-\frac{4}{h}\tilde{E}\tilde{Q}_{12}^T+\tilde{P}_{22}$$

$$\tilde{\Psi}_{15}=\frac{6}{h^2}\tilde{E}\tilde{Q}_{12}^T$$

$$\tilde{\Psi}_{16}=\tilde{X}^T\tilde{E}_A^T$$

$$\tilde{\Psi}_{17}=\tilde{X}^T\tilde{E}_h^T$$

$$\tilde{\Psi}_{22}=-\tilde{R}_1-\frac{4}{h}\tilde{E}\tilde{Q}_{22}\tilde{E}^T+\mu_1\tilde{X}^T\tilde{A}_h^T+\mu_1\tilde{A}_h\tilde{X}+\varepsilon_2\tilde{H}_h\tilde{H}_h^T$$

$$\tilde{\Psi}_{23}=-\mu_1\tilde{X}^T+\mu_2\tilde{A}_h\tilde{X}$$

$$\tilde{\Psi}_{24}=\frac{6}{h^2}\tilde{E}\tilde{Q}_{22}\tilde{E}^T-\frac{2}{h}\tilde{E}\tilde{Q}_{12}^T-\tilde{P}_{22}$$

$$\tilde{\Psi}_{25}=\frac{6}{h^2}\tilde{E}\tilde{Q}_{12}^T$$

$$\tilde{\Psi}_{33}=h\tilde{Q}_{22}-\mu_2\tilde{X}^T-\mu_2\tilde{X}$$

$$\tilde{\Psi}_{34}=\tilde{P}_{12}$$

$$\tilde{\Psi}_{35}=0$$

$$\tilde{\Psi}_{44}=\frac{6}{h^2}\left(\tilde{E}\tilde{Q}_{12}^T+\tilde{Q}_{12}\tilde{E}^T\right)-\frac{4}{h}\tilde{Q}_{11}-\frac{12}{h^3}\tilde{E}\tilde{Q}_{22}\tilde{E}^T$$

$$\tilde{\Psi}_{45}=\frac{6}{h^2}\tilde{Q}_{11}-\frac{12}{h^3}\tilde{E}\tilde{Q}_{12}^T$$

$$\tilde{\Psi}_{55}=-\frac{12}{h^3}\tilde{Q}_{11},$$

$$\tilde{X}=\begin{bmatrix}X_1 & 0\\ 0 & X_2\end{bmatrix},\quad \tilde{A}\tilde{X}=\begin{bmatrix}AX_1+BY & Y_LC\\ 0 & AX_2-Y_LC\end{bmatrix},$$

$$\tilde{A}_h\tilde{X}=\begin{bmatrix}A_hX_1 & 0\\ 0 & A_hX_2\end{bmatrix},\quad \tilde{E}_A\tilde{X}=\begin{bmatrix}E_1X_1 & E_1X_2\\ E_3Y & 0\end{bmatrix},$$

$$\tilde{E}_h\tilde{X}=\begin{bmatrix}E_2X_1 & E_2X_2\end{bmatrix},\quad X_2=V\hat{X}_2V^T,\quad X_2=\begin{bmatrix}\hat{X}_{11} & 0\\ \hat{X}_{21} & X_{22}\end{bmatrix}.$$

In addition, the controller gain and the observer gain in (24) are given $K=YX_1^{-1}$ and $L=Y_LUS\hat{X}_{11}^{-1}S^{-1}U^T$.

Proof: Under the conditions of the theorem, it follows from $\tilde{\Psi}_{33}<0$ that $\tilde{X}$ is nonsingular. Thus, X_2 is also nonsingular. Setting $Y_L=LUS\hat{X}_{11}S^{-1}U^T$ and $Y=KX_1$.

Since C is of full-row rank, that is to say $rank(C)=q$, its singular value decomposition can be given of the following form

$$C=U\begin{bmatrix}S & 0\end{bmatrix}V^T \tag{35}$$

where $S=diag\{s_1,s_2,\ldots,s_q\}$ and $s_i\left(i=1,\ldots,q\right)$ are nonzero singular values of C.

Hence,

$$\begin{aligned}Y_LC&=LUS\hat{X}_{11}S^{-1}U^TU\begin{bmatrix}S & 0\end{bmatrix}V^T\\ &=LU\begin{bmatrix}S\hat{X}_{11} & 0\end{bmatrix}V^T\\ &=LU\begin{bmatrix}S\hat{X}_{11} & 0\end{bmatrix}V^TV\begin{bmatrix}\hat{X}_{11} & 0\\ \hat{X}_{21} & X_{22}\end{bmatrix}V^T\\ &=LCX_2\end{aligned}$$

Thus, the augmented matrices can be written as

$$\tilde{A}\tilde{X}=\begin{bmatrix}AX_1+BKX_1 & LCX_2\\ 0 & AX_2-LCX_2\end{bmatrix},\quad \tilde{A}_h\tilde{X}=\begin{bmatrix}A_hX_1 & 0\\ 0 & A_hX_2\end{bmatrix},$$

$$\tilde{E}_A\tilde{X}=\begin{bmatrix}E_1X_1 & E_1X_2\\ E_3KX_1 & 0\end{bmatrix},\qquad \tilde{E}_h\tilde{X}=\begin{bmatrix}E_2X_1 & E_2X_2\end{bmatrix}.$$

Then, from theorem 3, the closed-loop singular system (24) is admissible.

Remark 5: For singular time-delay systems, the observer gain and controller gain are derived by solving LMIs. Since the regular time-delay systems area is a particular case of singular systems. Then, Theorem 3 is applicable to both singular time-delay systems and regular time-delay systems.

Remark 6: When the scalar parameters μ_1 and μ_2 are fixed, the condition (32) of Theorem 3 becomes LMI. However, choosing arbitrary μ_1 and μ_2 does not lead to the best result. In the following, a tuning procedure for the parameters μ_1 and μ_2 is proposed to enlarge the bound h_m on the time varying delay. If we select as optimization parameters μ_1 and μ_2 and choose a cost function $t_{\min}$, with $\tilde{\Psi}<t_{\min}I$, where $\tilde{\Psi}$ is defined in (32), then if there exists a combination of parameters μ_1 and μ_2 that gives a negative $t_{\min}$, these parameters give a feasible solution of the

conditions in Theorem 3 (finding this combination can be carried out by solving the corresponding feasibility problem). Finally, applying a numerical optimization algorithm, it is possible to obtain a locally convergent solution to the problem (using, e.g. *fminsearch* in the Optimization Toolbox [31]). If the resulting minimum value of the cost function is negative, then a combination of tuning parameters that solves the problem is found. This procedure can be derived as follows:

Algorithm 1: (finding μ_1 and μ_2 that maximize h_m)
Step 1: Fix initial values $\mu_1 = \mu_{10}$, $\mu_2 = \mu_{20}$ and $h_m = h_{m0}$, where h_{m0} must be small enough to have a feasible solution, and set a step variation h_{mstep}.
Step 2: Solve the following problem:

$$\min_{\mu_1,\mu_2} t_{\min} \quad such\ that \quad \tilde{\Psi} < t_{\min} \mathrm{I} \tag{36}$$

and obtain new values of μ_1 and μ_2.
Step 3: If $t_{\min} > 0$, the previous values of μ_1 and μ_2 give the largest bound of delay; otherwise $(t_{\min} \le 0)$, to improve the solution, set $h_m = h_m + h_{mstep}$ and repeat from Step 2.

Remark 7: To apply our method to a real system, we need to use other components, for example the interfaces and also cablings. These added components may have influences on the desired performance. To overcome this, we must take into account the parameters of these components to build the effective control law. Also, as is well known, in the case of large-scale problems, the dimension of the matrix inequality becomes large, this will increase the time complexity.

Numerical examples

In this section, we give two examples to illustrate the effectiveness of our results.

Example 1: Consider the singular time-delay system (5) with

$$E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad A = \begin{bmatrix} -0.3012 & 0.1257 \\ 0.2351 & -1.0998 \end{bmatrix},$$
$$A_h = \begin{bmatrix} -0.5c & 0 \\ 0 & -0.1c \end{bmatrix}.$$

where c is a scalar parameter.

To compare with our results, we compute the time-delay upper bound $\bar{h}$ by Theorem 1 and other results of [22,23,24,25,26,27], the results are summarized in Table 1. From Table 1 we can see that our results are less conservative than those reported in the above-mentioned papers.

Example 2: Consider the uncertain singular time-delay system (1) with the following parameters

$$E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad A = \begin{bmatrix} 1.5 & 0.4 & 0.2 \\ 0.3 & -0.6 & 0.4 \\ 0.9 & 0.5 & 0.8 \end{bmatrix},$$
$$A_h = \begin{bmatrix} 0.4 & -0.3 & 0.2 \\ 0.2 & 0.3 & 0.4 \\ 0.1 & 0.5 & -0.2 \end{bmatrix}, B = \begin{bmatrix} 0.5 \\ 1.5 \\ 1 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix},$$
$$H = \begin{bmatrix} 1.5 \\ 1.5 \\ 1.5 \end{bmatrix}, E_1 = \begin{bmatrix} 0 & 0 & 0.5 \end{bmatrix}, E_2 = \begin{bmatrix} 0 & 0 & 0.3 \end{bmatrix}, E_3 = 0.3.$$

Let $\bar{h} = 1.5, \varepsilon_1 = 1, \varepsilon_2 = 0.01$ and $\mu_1 = \mu_2 = 0.01$, solving the LMIs obtained from Theorem 3 by using Toolbox in MATLAB (MathWorks, Natick, MA, USA). The controller matrix and observer gain matrix are designed as

$$K = \begin{bmatrix} -4.0193 & 0.7310 & -74.4440 \end{bmatrix},$$
$$L = \begin{bmatrix} -7.1225 & 16.0217 \\ -116.9783 & 134.3485 \\ -87.9241 & 100.2778 \end{bmatrix}.$$

Applying the theorem 3.3 in [22], the followings controller gain matrix K and the observer gain matrix L are obtained.

$$K = \begin{bmatrix} -5.0835 & 0.9470 & -96.4382 \end{bmatrix},$$
$$L = \begin{bmatrix} -17.1792 & 27.8765 \\ -172.3307 & 195.0774 \\ -129.6115 & 146.1288 \end{bmatrix}.$$

A simple comparison between the gains shows the improvement brought by the proposed methodology.

Given the initial conditions $\phi(t) = \begin{bmatrix} -0.05 & 0.65 & 1 \end{bmatrix}^T$; $t \in [-1.5, 0]$, and $F(t) = 0.8 + 0.2\sin(t)$ the simulation results are presented in Figure (1-6). Figure (1-3) shows the states of the closed-loop system, and Figure (4-6) depicts the errors dynamic system, respectively. From the figures, it is quite clear that the generated control law guarantees regulation to the zero level, and our results are less conservative than those reported in [22].

Table 1. Comparison of maximum allowed delays $\overline{h}$ for example 1.

c	1	1.2	1.4	1.6	2
Fridman et al [23]	2.0362	1.7691	1.5619	1.3977	1.1548
Xu et al [24]	2.2750	1.9635	1.7282	1.5438	1.2729
Ding et al [25]	4.1749	3.1753	2.5725	2.1659	1.6491
Feng Y et al [26] (m=2)	4.8587	3.6397	2.9197	2.4413	1.8419
Feng Z et al [27](m=2)	4.8608	3.6418	2.9220	2.4436	1.8443
Kchaou et al [22](m=2)	4.9173	3.6770	2.9467	2.4623	1.8565
Theorem 1	5.0645	3.7819	3.0276	2.5278	1.9037

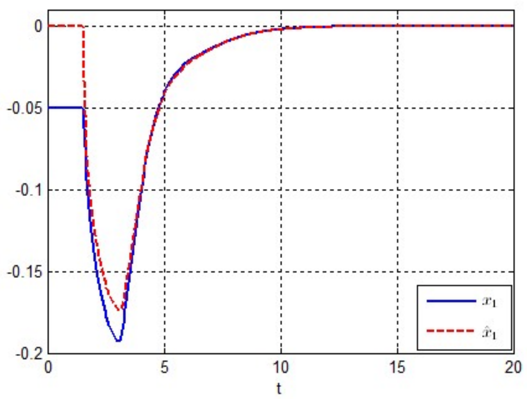

Figure 1. Response and estimate of the state x_1.

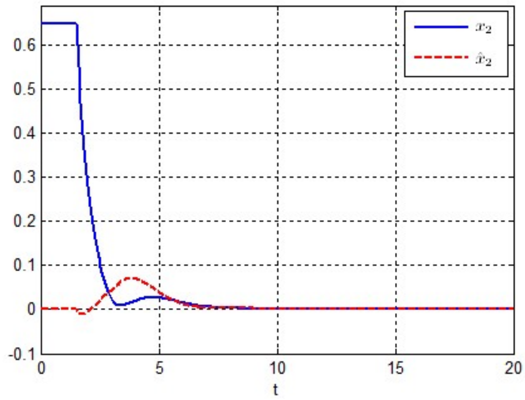

Figure 2. Response and estimate of the state x_2.

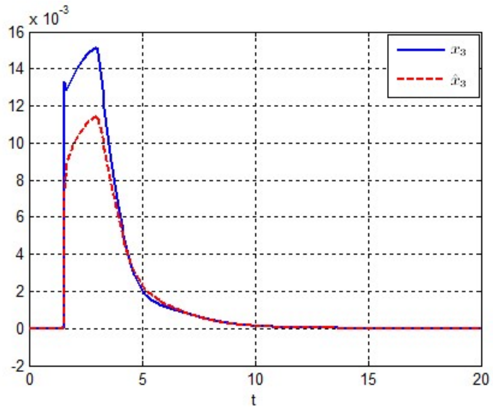

Figure 3. Response and estimate of the state x_3.

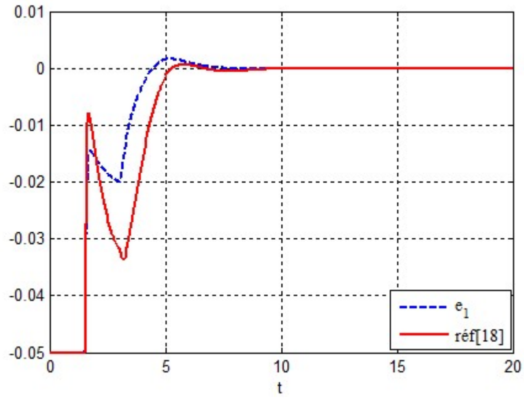

Figure 4. Trajectory of error state e_1.

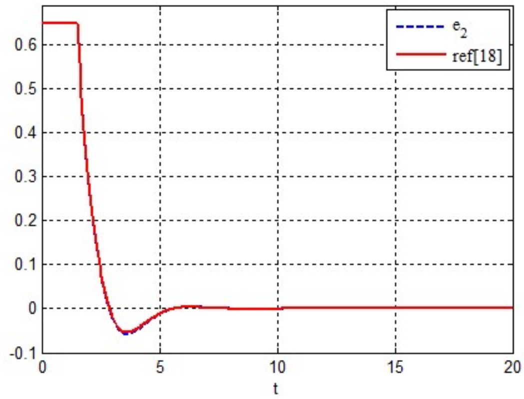

Figure 5. Trajectory of error state e_2.

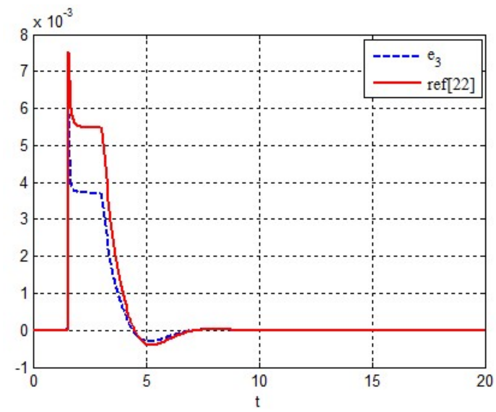

Figure 6. Trajectory of error state e_3.

Conclusion

In this paper, the problem of robust output feedback control for linear delay singular systems with uncertainties is addressed. First, a delay-dependent stability criterion is derived for the nominal system (i.e without uncertainties). The proposed results are less conservative than some existing ones. Then, based on the obtained condition, sufficient conditions for admissibility and the design of observer gain and the observer-based controller gain are derived for uncertain systems. Finally, numerical examples and simulation results are given to demonstrate the validity of the proposed method.

References

[1] L. Dai, "Singular control systems," New York: Springer-Verlag, 1989.

[2] J. K. Hale and SM. Verduvn Lunel, "Introduction to functional differential equations," New York, NY: Springer-Verlag, 1993.

[3] N. Chaibi, E. H. Tissir, and A. Hmamed, "Delay Dependent Robust Stability of Singular Systems with Additive Time-Varying Delays," *International Journal of Automation and Computing*, vol. 10, no. 1, pp. 85-90, 2013.
doi: 10.1007/s11633-013-0700-6

[4] Y. Wang, Q. Wang, P. Zhou, and D. Duan, "Robust guaranteed cost control for singular Markovian jump systems with time-varying delay," *ISA Trans*, vol. 51, no. 5, pp. 559-65, 2012.
doi: 10.1016/j.isatra.2012.04.005

[5] S. Wo, Y. Zou, Q. Chen, and S. Xu, "Non fragile controller design for discrete descriptor systems," *J Frankl Inst*, vol. 346, no. 9, pp. 914-22, 2009.
doi: 10.1016/j.jfranklin.2009.07.008

[6] S.-Y. Xu, J. Lam, and C.-W. Yang, "Robust H_∞ control for uncertain singular systems with state delay," *International Journal of Robust Nonlinear Control*, vol. 13, pp. 1213-1223, 2003.
doi: 10.1002/rnc.837

[7] M. Fang, "Delay-dependent robust H_∞ control for uncertain singular systems with state delay," *Acta Automatica Sinica*, vol. 35, no. 1, pp. 65-70, 2009.
doi: 10.1016/S1874-1029(08)60066-X

[8] L. Li and Y. Jia, "Observer-based resilient $L_2 - L_\infty$ control for singular time-delay systems," *IET Control Theory and Applications*, vol. 3, no. 10, pp. 1351-1362, 2009.
doi: 10.1049/iet-cta.2008.0361

[9] N. Chaibi, E. H. Tissir, and A. Hmamed, "Delay dependent exponential stability and guaranteed cost of time-varying delay singular systems," *Applied Mathematical Sciences,* vol. 6, no. 114, pp. 5655-5666, 2012.

[10] E. H. Tissir, "Delay dependent robust stabilization for systems with uncertain time varying delays," *Journal of Dynamic Systems, Measurement and control*, vol. 132, 054504-1-054504-7, 2010.
doi: 10.1115/1.4002066

[11] P. Park, J.-W. Ko, C. Jeong, "Reciprocally convex approach to stability of systems with time-varying delays," *Automatica,* vol. 47, no. 1, pp. 235-238, 2011.
doi: 10.1016/j.automatica.2010.10.014

[12] LD. Guo, H. Gu, J. Xing, XQ. He, "Asymptotic and exponential stability of uncertain system with interval delay," *Applied Mathematics and Computation*, vol. 218, no. 19, pp. 9997-10006, 2012.
doi: 10.1016/j.amc.2012.03.074

[13] W. Qian, J. Liu, "New stability analysis for systems with interval time-varying delay," *Journal of the Franklin Institute*, vol. 350, no. 4, pp. 890-897, 2013.
doi: 10.1016/j.jfranklin.2012.12.017

[14] A. Seuret, F. Gouaisbaut, "Wirtinger-based integral inequality: application to time-delay systems," *Automatica*, vol. 49, no. 9, pp. 2860-2866, 2013.
doi: 10.1016/j.automatica.2013.05.030

[15] J. Reilly, "Observers for Linear Systems," *Academic,* New York, 1983.

[16] P.-L. Liu, "Observer-based Control for Time- varying Delay Systems with Delay-dependence," *Universal Journal of Electrical and Electronic Engineering*, vol. 1, no. 2, pp. 31-40, 2013.
doi: 10.13189/ujeee.2013.010204

[17] F. El Haoussi and E. H. Tissir, "Robust H_∞ Controller Design for Uncertain Neutral Systems via Dynamic Observer Based Output Feedback," *International Journal of Automation and Computing*, vol. 6, no. 2, pp. 164-170, 2009.
doi: 10.1007/s11633-009-0164-x

[18] N. El Fezazi, F. El Haoussi, E. H. Tissir, and F. Tadeo, "Delay dependent anti-windup synthesis for time varying delay systems with saturating actuators," *International Journal of Computing Application*, vol. 111, no. 1, pp. 1-6, 2015.
doi: 10.5120/19499-1107

[19] N. El Fezazi, F. El Haoussi, E. H. Tissir, A. R. Husain and M. I. Zakaria, "Delay dependent anti-windup synthesis approach to AQM in TCP/IP Networks," in proceeding of 2^{nd} *International Conference on Electrical and Information Technologies*, Tangiers, Morocco, May 4-7, 2016.
doi: 10.1109/EITech.2016.7519609

[20] M. Darouach, L. Boutat-Baddas, M. Zerrougui, " H_∞ observers design for a class of nonlinear singular systems," *Automatica,* vol. 47, no. 11, pp.2517-2525, 2011.
doi: 10.1016/j.automatica.2011.08.037

[21] L. Qin, Z. Qingling, Z. Yanjuan, A. Yichun, "Observer-based passive control for descriptor systems with time-delay," *Journal of Systems Engineering and Electronics*, vol. 20, no. 1, pp. 120-128, 2009.

[22] M. Kchaou, H. Gassara, A. El-Hajjaji, "Robust observer-based control design for uncertain singular systems with time-delay," *International Journal of Adaptive Control and Signal Processing,* vol. 28, pp. 169-183, 2014.
doi:10.1002/acs.2409

[23] E. Fridman and U. Shaked, " H_∞ control of linear state-delay descriptor systems: an LMI approach," *Linear Algebra and Its Applications*, vol. 351-352, pp. 271-302, 2002.
doi: 10.1016/S0024-3795(01)00563-8

[24] S. Xu, PV. Dooren, R. Stefan, and J. Lam, "Robust stability and stabilization for singular systems with state delay and parameter uncertainty," *IEEE Transactions on Automatic Control*, vol. 47, no. 7, pp. 1122-1128, 2002.
doi: 10.1109/TAC.2002.800651

[25] Y. Ding, S. Zhong, and W. Chen, "A delay-range-dependent uniformly asymptotic stability criterion for a class of nonlinear singular systems," *Nonlinear Analysis: Real World Applications*, vol. 12, no. 2, pp. 1152-1162, 2011.
doi: 10.1016/j.nonrwa.2010.09.009

[26] Y. Feng, X. Zhu, and Q. Zhang, "Delay-dependent stability criteria for singular time-delay systems," *Acta Automatica Sinica,* vol. 36, no. 3, pp. 433-437, 2010.
doi: 10.1016/S1874-1029(09)60019-7

[27] Z. Feng, J. Lam, and H. Gao, " α -dissipativity analysis of singular time-delay systems," *Automatica,* vol. 47, no. 11, pp.2548-2552, 2011.
doi: 10.1016/j.automatica.2011.06.025

[28] E. H. Tissir, "Delay dependent robust stability of linear systems with non-commensurate time varying delays," *International Journal of Systems Sciences*, vol. 38, pp.749-757, 2007.
doi: 10.1080/00207720701597415

[29] X. Li and C. E. de Souza, "Criteria for Robust Stability and Stabilization of Uncertain Linear Systems with State Delay," *Automatica,* vol. 33, pp. 1657-1662, 1997.
doi: 10.1016/S0005-1098(97)00082-4

[30] P. Finsler, "Ȕber das vorkommen definiter semi-definiter formen is scharen quadratischer formen," *Commentarii mathematici hevetici,* vol. 9, pp. 188-192, 1937.

[31] T. Coleman, M. Branch, and A. Grace, "Optimization Toolbox for Use with Matlab," *MathWorks*, Natick, Mass, USA, 1999.