ORIGINAL ARTICLE SPECIAL ISSUE: Intelligent Robotics(2/2) Guest edited by Yi-Hung Liu

Sequential Quadratic Method for GPS NLOS Positioning in Urban Canyon Environments

He-Sheng Wang[1,*], Chen-Ying Kao[2], and Jing-Fen Chen[1]

[1]*Department of Communications, Navigation, and Control Engineering, National Taiwan Ocean University, Taiwan*

[2]*Service Systems Technology Center, Industrial Technology Research Institute, Taiwan*

(**Received** 5 October2012; **Accepted** 25 October 2012; **Published on line** 1 March 2013)

*Corresponding author: hswang@mail.ntou.edu.tw

DOI: 10.5875/ausmt.v3i1.166

Abstract: In this paper, the problem of GPS non-line-of-sight (NLOS) positioning in urban canyon environments is considered. We propose a new position-determination estimator based on sequential quadratic programming (SQP) that is able to estimate and eliminate the path-delay error caused by the indirect transmission of the GPS signal. The estimator takes into account the measurement bias resulting from NLOS transmission and also improves the location accuracy of the satellite positioning system. The present method can effectively eliminate NLOS delay errors and improves the location accuracy of a satellite navigation receiver in an urban canyon environment. A Wilcoxon-norm-based regressor is further derived to improve the probability of detection of NLOS biases. The Wilcoxon regressor is a robust estimator that is well-suited to identifying outliers (in our case, NLOS biases) during the regression process. Experimental results demonstrate that the proposed estimator can accurately compute a user's location after identifying and removing the measurement biases.

Keywords: Non-Line-of-Sight; Sequential Quadratic Programming; Global Positioning System; Wilcoxon Regression; Urban Canyon Environment

---

1. Introduction

PNDs (Personal Navigation Devices) have become nearly ubiquitous. With the help of satellite navigation systems (such as GPS), an off-the-shelf PND can provide real-time positioning information accurate to within a few meters. As the field of PND applications expands, new navigation devices are subject to problems not considered in the original system design. For example non-line of sight (NLOS) reception of the navigation signal may drastically degrade the quality of the positioning result. Although NLOS problems were acknowledged when GPS products were first introduced, the issue is significantly more pronounced in urban canyon environments, and urban PND applications were not widely anticipated when GPS was first invented. Among the key issues faced in urban navigation is multipath error caused by signal reflections, diffractions, and scatterings. For satellite navigation, this and ionosphere propagation delay are the main sources of location determination error.

Recently, non-GPS based wireless positioning technologies have emerged as a major means for ground-based navigation where GPS signals are weak or not available (or at least is degraded dramatically) [1]. Ground-based wireless positioning is more vulnerable to multipath interference than satellite-based positioning system and this has prompted the development of many multipath mitigation techniques for ground-based wireless positioning systems. Methods for averaging measurement sequences are introduced in [2] and [3].

Copyright © 2013 International Journal of Automation and Smart Technology

Specifically, [2] proposes LOS reconstruction, while [3] uses a residual weighting method. Both the LOS reconstruction and the residual weighting methods are capable of reducing the effect of the fast fading noise error caused by multipath errors. Other methods have been developed to improve location accuracy based on *a priori* statistics or empirical databases of NLOS errors are derived (e.g., [4, 5]. Another method for mitigating the NLOS effect is constrained optimization. When there is only limited information concerning measurement biases, constrained optimization is particularly useful for estimating desirable parameters [6]. In particular, in [7] a three-step method based on interior-point optimization is used to estimate and eliminate the measurement bias contained in each pseudorange. The abovementioned methods have their own ground in the field of wireless positioning. However, these concepts have seldom been applied to the field of satellite-based positioning system (such as GPS). The present paper adapts the method of constrained optimization in [7] to develop a new position estimator for GPS receivers.

This paper aims to provide a method to efficiently estimate and eliminate the path-delay error. A new position-determination estimator is proposed based on sequential quadratic programming (SQP). The estimator takes into account the measurement bias resulting from NLOS transmission and also improves the location accuracy of satellite positioning systems. The present method effectively eliminates NLOS (non-line-of-sight) delay errors and also improves the location accuracy of satellite navigation receivers. A Wilcoxon-norm-based regressor is further derived to improve the probability of detection of NLOS biases. The Wilcoxon regressor is a robust estimator that is well-suited for identifying outliers (in our case, NLOS biases) during the regression process. Experimental results demonstrate that, after identifying and removing measurement biases, the proposed estimator can accurately compute a user's location.

The rest of this paper is organized as follows: In Section 2, the method of SQP optimization is briefly described. In Section 3, the proposed method for GPS NLOS detection is introduced. The SQP algorithm can accurately estimate the bias error, but is not well-suited for identifying the satellite signal which actually contains the NLOS error. Therefore, we introduce a robust nonlinear regression using the Wilcoxon norm to help identify the NLOS propagation prior to using the SQP method, so as to reduce the number of constraint equations and improve positioning accuracy. Experimental results are given in Section 4, and some concluding remarks are provided in Section 5.

He-Sheng Wang was born in Taiwan in 1970. He received the B.S. degree in Electrical Engineering from National Central University in 1993, and the M.S. and Ph.D. degrees in Electrical Engineering from National Taiwan University in 1995 and 1999, respectively. He is currently a faculty member at Department of Communications, Navigation and Control Engineering, National Taiwan Ocean University. His research interests include H∞ control, linear and nonlinear control of descriptor systems, Differential Geometry and Its Applications in Navigation and Control, and mathematical theory of satellite navigation.

Chen-Ying Kao was born in Taiwan in 1960. He received the B.S. degree in Electrical Engineering from National Taiwan Ocean University in 1984 and the M.S. degree in Software Engineering from Azusa Pacific University in USA in 1993, respectively. She is currently a project manager at Service Systems Technology Center of Industrial Technology Research Institute. Her research interests include Trajectory Analysis of Moving Object, Dynamic Background Stitching using GNSS Heterogeneous Data Fusion, and Development Cold Chain Logistics Tracking Platform using Cloud Computing Technology.

Jing-Fen Chen was born in Taiwan in 1989. She received the B.S. degree in Department of Communications, Navigation and Control Engineering from National Taiwan Ocean University in 2011. She is currently a Graduate Student at Department of Communications, Navigation and Control Engineering, National Taiwan Ocean University. Her present research subject is the observability analysis of navigation system using a geometric approach.

2. Sequential Quadratic Programming

The method of sequential quadratic programming was first proposed by R. B Wilson in 1963 to solve constrained nonlinear optimizations [8]. It relies on a profound theoretical foundation and provides powerful algorithmic tools to solve large-scale technologically relevant problems.

Consider the following nonlinear optimization problem:

$$\begin{aligned} &\min f(x), \\ &\text{subject to } \mathbf{h}(\mathbf{x}) = 0, \\ &\qquad\qquad\quad \mathbf{g}(\mathbf{x}) \le 0, \end{aligned} \tag{1}$$

where $\mathbf{x}$ is an n-by-1 vector, $\mathbf{h}(\mathbf{x})$ is a p-by-1 vector of equality constraint, and $\mathbf{g}(\mathbf{x})$ is a q-by-1 vector of inequality constraint. To solve the constrained optimization problem, we can define the following Lagrangian functional:

$$L(\mathbf{x},\boldsymbol{\lambda},\boldsymbol{\tau}) = f(\mathbf{x}) + \boldsymbol{\lambda}^T\mathbf{h}(\mathbf{x}) + \boldsymbol{\tau}^T\mathbf{g}(\mathbf{x}), \tag{2}$$

where $\boldsymbol{\lambda}$ and $\boldsymbol{\tau}$ are the Lagrangian multipliers. Assume Equation (2) is first-order continuously differentiable, and suppose there exists a local minimum at $\mathbf{x}^*$, then the following first-order Karush-Kuhn- Tucker (KKT) condition holds:

$$\nabla L(\mathbf{x}^*,\boldsymbol{\lambda},\boldsymbol{\tau}) = \nabla f(\mathbf{x}) + \nabla\mathbf{h}(\mathbf{x}^*)\boldsymbol{\lambda} + \nabla\mathbf{g}(\mathbf{x}^*)\boldsymbol{\tau} = 0. \tag{3}$$

If we further assume that Equation (2) is second-order continuously differentiable, then the following second-order KKT conditions also hold:

Copyright © 2013 International Journal of Automation and Smart Technology

(i) $\boldsymbol{\lambda} \geq 0, \boldsymbol{\tau} \geq 0, \boldsymbol{\tau}^T(i)\boldsymbol{\lambda}(i) = 0, \forall i$

(ii) The columns of the matrix $\mathbf{G}(\mathbf{x}^*)$ are linearly independent, where $\mathbf{G}(\mathbf{x}^*)$ is defined as:

$$\mathbf{G}(\mathbf{x}^*) \equiv \left[\nabla h_1(\mathbf{x}^*) \cdots \nabla h_p(\mathbf{x}^*)\ \nabla g_1(\mathbf{x}^*) \cdots \nabla g_q(\mathbf{x}^*)\right].$$

(iii) $\mathbf{d}^T\mathbf{H}_L(\mathbf{x}^*, \boldsymbol{\lambda}, \boldsymbol{\tau})\mathbf{d} > \mathbf{0}$, where $\mathbf{H}_L(\mathbf{x}^*, \boldsymbol{\lambda}, \boldsymbol{\tau})$ is the Hessian matrix of the Lagrangian functional, and $\mathbf{d}$ is a vector satisfying $\mathbf{G}^T(\mathbf{x}^*)\mathbf{d} > 0$.

If conditions (i)~(iii) hold, then $\mathbf{x}^*$ is the absolute global minimum. The SQP method first considers the equality constraint, and then transforms the original function to a quadratic optimization problem with inequality constraint. To this end, consider Problem (1) with only the equality constraint $\mathbf{h}(\mathbf{x}) = 0$, and apply the Newton-Raphson method. Then, at the k-th iteration, we have the following linearized equation:

$$\begin{bmatrix} \mathbf{x}_{k+1} \\ \boldsymbol{\lambda}_{k+1} \end{bmatrix} - \begin{bmatrix} \mathbf{x}_k \\ \boldsymbol{\lambda}_k \end{bmatrix} = \begin{bmatrix} \mathbf{H}_L(\mathbf{x}_k, \boldsymbol{\lambda}_k) & \nabla \mathbf{h}(\mathbf{x}_k) \\ \nabla \mathbf{h}^T(\mathbf{x}_k) & 0 \end{bmatrix}^{-1} \begin{bmatrix} -\nabla f(\mathbf{x}_k) \\ -\mathbf{h}(\mathbf{x}_k) \end{bmatrix}.$$

Upon transposing the above equation and making a change of variables of the form $\mathbf{s} = \mathbf{x}_{k+1} - \mathbf{x}_k$ and $\mathbf{u} = \lambda_{k+1} - \lambda_k$, we then arrive at the following equation:

$$\begin{bmatrix} \mathbf{H}_L(\mathbf{x}_k, \boldsymbol{\lambda}_k) & \nabla \mathbf{h}(\mathbf{x}_k) \\ \nabla \mathbf{h}^T(\mathbf{x}_k) & 0 \end{bmatrix} \begin{bmatrix} \mathbf{s} \\ \mathbf{u} \end{bmatrix} = \begin{bmatrix} -\nabla f(\mathbf{x}_k) \\ -\mathbf{h}(\mathbf{x}_k) \end{bmatrix}. \tag{4}$$

The above equation can be thought of as a sub-problem of the form:

$$\begin{aligned} &\min \nabla f^T(\mathbf{x}_k)\mathbf{s} + \frac{1}{2}\mathbf{s}^T\mathbf{H}_L(\mathbf{x}_k, \boldsymbol{\lambda}_k)\mathbf{s}, \\ &\text{subject to } \mathbf{h}(\mathbf{x}_k) + \nabla \mathbf{h}^T(\mathbf{x}_k)\mathbf{s} = \mathbf{0}, \end{aligned} \tag{5}$$

which has the same KKT condition with the original problem, i.e., Equation (1); and finally by some intricately mathematical proof, we know that the inequality constraint will have a similar form as the equality constraint. Therefore we can construct a quadratic programming sub-problem as follows:

$$\begin{aligned} &\min \nabla f^T(\mathbf{x}_k)\mathbf{s} + \frac{1}{2}\mathbf{s}^T\mathbf{H}_L(\mathbf{x}_k, \boldsymbol{\lambda}_k)\mathbf{s}, \\ &\text{subject to } \mathbf{h}(\mathbf{x}_k) + \nabla \mathbf{h}^T(\mathbf{x}_k)\mathbf{s} = \mathbf{0}, \\ &\qquad\qquad \mathbf{g}(\mathbf{x}_k) + \nabla \mathbf{g}^T(\mathbf{x}_k)\mathbf{s} \leq 0. \end{aligned} \tag{6}$$

In [8], it was shown that, assuming the original nonlinear optimization problem is convex and satisfies all the necessary KKT conditions, the solution of each sub-problem (6) exists. If the quadratic programming sub-problem is well convergent, the optimal solution of nonlinear problem (1) can be obtained by iteratively repeating this process. Two additional properties of the SQP method should be pointed out. First, SQP is not a feasible point method, i.e., neither the initial point nor any of the subsequent iterates need to be feasible (where a feasible point satisfies all of the constraints of the nonlinear problem). This is a major advantage since finding a feasible point when there are nonlinear constraints may be nearly as hard as solving the nonlinear programming problem itself. Second, the success of the SQP method depends on the existence of rapid and accurate algorithms to solve quadratic programs.

3. NLOS Detection and Elimination by SQP Method

The SQP algorithm can accurately estimate the bias error, but it is not well-suited for identifying the satellite signal which actually contains an NLOS error. Therefore, we introduce a robust nonlinear regression using the Wilcoxon norm to help identify the NLOS propagation prior to using the SQP method, so as to reduce the number of constraint equations and improve positioning accuracy. Next, we first review some basic facts concerned with GPS.

GPS is a satellite-based navigation system consisting of a network of 24 satellites placed into six mid-earth orbits. GPS satellites circle the earth twice a day in a precise orbit and transmit signals modulated by navigation messages to users on the ground. The basic idea of GPS positioning is that users then consider this information and use multi-lateration to calculate the GPS receiver's exact solution. Essentially, after receiving the satellite signal information, the GPS user is able to construct the basic pseudorange measurement equation as follows:

$$\begin{aligned} &\rho^{(i)}(t) = \\ &r(t, t-\tau) + c[\delta t_u(t) - \delta t^{(i)}(t-\tau)] + I^{(i)}(t) + T^{(i)}(t) + \varepsilon^{(i)}(t) \\ &r(t, t-\tau) \triangleq \\ &\sqrt{(x^{(i)}(t-\tau) - x_u(t))^2 + (y^{(i)}(t-\tau) - y_u(t))^2 + (z^{(i)}(t-\tau) - z_u(t))^2}, \end{aligned} \tag{7}$$

where $i = 1, 2, \ldots, K$ is the number of satellites in view; $r^{(i)}(t, t-\tau)$ is the actual distance between the receiver at signal reception time t, and the satellite at signal transmission time $t-\tau$; $\mathbf{s}^i := (x^{(i)}, y^{(i)}, z^{(i)})$ is the i-th satellite's coordinate; $\mathbf{u} := (x_u, y_u, z_u)$ is the user's position coordinate; c is the speed of light; $\delta t_u(t)$ and $\delta t^{(i)}(t-\tau)$ respectively are the receiver and satellite clock offsets relative to GPS time; $I^{(i)}(t)$ and $T^{(i)}(t)$ respectively are the ionospheric and

Copyright © 2013 International Journal of Automation and Smart Technology

tropospheric propagation delays; and $\varepsilon^{(i)}(t)$ accounts for modeling errors and unmodeled effects (such as thermal noise and multipath). Because of the offsets and propagation delays, the ranges to GPS satellites measured by a receiver are called pseudoranges. Users would correct each measured pseudorange for the known errors for parameter values in the navigation message from the satellite. The main corrections available to a civil user are the satellite clock offset, the ionospheric delay using the parameter values for the Klobuchar model, and the software model accounting for the tropospheric delays. $\rho_c^{(i)}$ is the pseudorange obtained after accounting for various offset and error terms. We thus rewrite Equation (7) for the corrected pseudorange measurement as follows:

$$\begin{aligned}\rho_c^{(i)} &= r^{(i)} + c\cdot\delta t_u + \tilde{\varepsilon}^{(i)} \\ &= r^{(i)} + b + \tilde{\varepsilon}^{(i)},\end{aligned} \tag{8}$$

where $\tilde{\varepsilon}^{(k)}$ denotes the combined effect of the residual errors. For simplicity, in Equation (8) we have dropped explicit reference to time. Equation (8) has four unknowns, namely the user's position (x_u, y_u, z_u) and the clock bias δt_u. Hence, if we have at least four pseudorange measurements, Equation (8) can be linearized and then solved by least-squares method to obtain the user's coordinates. This procedure can be found in many advanced GPS textbooks (see e.g. [9]); here we have only emphasized how the satellite ephemeris is used in Equation (8). To solve the corrected pseudorange equation, the satellite's position ($x^{(i)}$, $y^{(i)}$, $z^{(i)}$) is assumed to be known. For traditional GPS receivers, the satellite's position can be computed using the broadcast ephemeris downloaded from the GPS navigation message. The ephemeris broadcast from the satellites themselves is valid only for 2 to 4 hours. The GPS control segment has a future ephemeris beyond the next 4 hours, and the ground stations upload this data to the satellites. This set of data consists of six Keplerian elements as well as a few correction terms. The only way to access this information is to wait for the ground station to update. On the other hand, to accurately compute the satellite position, the full set of ephemeris data must be obtained from the satellite signal, which, at a rate of 50 bps, usually requires at least 30 seconds of non-interrupted reception. However, urban GPS users may not be able to provide continuous reception due to signal blockage. Furthermore, even if the mobile device can continuously track the GPS signal, most users still expect their devices to determine their position within a few seconds. This raises a demand for some available source of future ephemeris that can be retrieved in a timely manner and has a relatively long expiration time.

In an urban-canyon environment, multipath interference usually results in large pseudorange errors which can be seen as outliers in the robust regression problem. The idea here is to introduce one extra bias parameter so as to reflect the path delay caused by NLOS transmission. Under this circumstance, Equation (8) is written as:

$$\rho(i) = \left\|\mathbf{u} - \mathbf{s}^i\right\| + b + m(i),$$

where $\rho(i)$ is the pseudorange measurement of i-th satellite; $\mathbf{u}$ is the user's position vector; $\mathbf{s}^i$ is the 3D coordinate of the i-th satellite; b is the receiver's clock bias; and $m(i)$ is the NLOS bias in the i-th pseudorange measurement. Note that the residual error term is omitted in the previous equation. To estimate the NLOS bias, we define the following nonlinear programming problem:

$$\begin{aligned}&\min f(\mathbf{u},b,m) = \sum_{i=1}^{n}\left(\rho(i) - \left\|\mathbf{u} - \mathbf{s}^i\right\| - b - m(i)\right)^2, \\ &\text{subject to } m(i) = 0, i \in LOS, \\ &\qquad 10 \le |m(i)| \le 200, i \in NLOS,\end{aligned}$$

where *LOS* denotes the set of satellites with line-of-sight transmission, and *NLOS* denotes the set of satellites having only a NLOS signal. By applying the procedure outlined in Section 2, we then obtain the following sub-problem at k-th iteration:

$$\begin{aligned}&\min \mathbf{A}_k^T\mathbf{s} + \frac{1}{2}\mathbf{s}^T\mathbf{B}_k\mathbf{s}, \\ &\text{subject to } m_k(i) + \mathbf{s}(i+4) = 0, i \in LOS, \\ &\qquad m_k(i) + \mathbf{s}(i+4) - 200 \le 0, i \in NLOS,\end{aligned}$$

where $\mathbf{A}_k = \nabla f(\mathbf{u},b,m)$, and $\mathbf{B}_k$ is the Hessian matrix of the following Lagrangian function

$$\begin{aligned}L(\mathbf{u},b,m,\boldsymbol{\lambda},\boldsymbol{\tau}) &= \sum_{i=1}^{n}\left(\rho(i) - \left\|\mathbf{u} - \mathbf{s}^i\right\| - b - m(i)\right)^2 \\ &+ \sum_{i_1=1}^{r_1}\boldsymbol{\lambda}(i_1)m(i_1) + \sum_{i_2=1}^{r_2}\boldsymbol{\tau}(i_2)\left(m(i_2) - 200\right) \\ &+ \sum_{i_2=r_2+1}^{2r_2}\boldsymbol{\tau}(i_2)\left(10 - m(i_2)\right)\end{aligned}$$

where r_1 denotes the amount of LOS satellites, r_2 denotes the number of NLOS satellites, and $r_1 + r_2 = n$. A detailed calculation of the matrices $\mathbf{A}_k$ and $\mathbf{B}_k$ is given in the appendix.

Copyright © 2013 International Journal of Automation and Smart Technology

4. Simulation and Experimental Results

This section describes several simulations and experiments conducted to evaluate the performance of the proposed method. Before giving the results of the simulation, we first introduce another technique to improve the probability of detection of NLOS transmission. Specifically, the method is based on the Wilcoxon regressor. Consider the Wilcoxon norm of the measured pseudorange:

$$\|\boldsymbol{\rho}\|_W := \sum_{i=1}^{n} a(R(\rho_i))\rho_i = \sum_{i=1}^{n} a(i)v_{(i)},$$
$$\boldsymbol{\rho} := \begin{bmatrix} \rho_1 & \cdots & \rho_n \end{bmatrix}^T \in \measuredangle^n,$$

where $R(\rho_i)$ denotes the order of ρ_i among the ρ_i's so that the outcome of $R(\cdot)$ is the ordered values $v_{(1)} \le \ldots \le v_{(n)}$; $a(i) := \varphi[i/(n+1)]$, and $\varphi(v) := \sqrt{12}(v-0.5)$ is the Wilcoxon score function. The Wilcoxon norm is a semi-norm in the sense that $\|\boldsymbol{\rho}\|_W = 0$ if and only if $\rho_1 = \rho_2 = \ldots = \rho_n$. Let the Wilcoxon estimator of **x** be defined as follows:

$$\hat{\mathbf{x}}_W = \arg\min_{\mathbf{x}} \|\boldsymbol{\rho} - \mathbf{r}\|_W,$$

where $\boldsymbol{\rho} = (\rho_1, \ldots, \rho_n), \mathbf{r} = (r_1, \ldots r_n)$. The next theorem gives an existence condition for the Wilcoxon estimator.

Theorem 1. Consider model (8). Suppose that **h**(**x**) is defined and continuous on a compact subset Θ of $\mathbf{R}^4$. Then the Wilcoxon estimator exists on Θ.

As pointed out earlier, in an urban-canyon environment, multipath interference usually results in large pseudorange errors which can be seen as outliers in the robust regression problem. Therefore, a pseudorange measurement contaminated by NLOS will tend to have a larger Wilcoxon score.

To evaluate the performance of the Wilcoxon estimator, a Monte-Carlo simulation of 1000 runs is conducted. For the Wilcoxon estimator, pseudorange measurements with NLOS biases usually have higher scores. For example, as shown in Table 1, the four satellites with SV Numbers 2, 5, 9, and 22 have higher scores, which agrees with our experimental data to which we have purposely added NLOS biases in these four satellites. In this case, we consider this run as successful. The result of the Monte-Carlo simulation is shown in Figure 1. We compare three different methods, namely the Wilcoxon estimator, hypothesis testing, and the residual-based method (traditional RAIM method). The hypothesis testing method is based on the procedure outlined in [6], and the residual-based method is called receiver autonomous integrity monitoring (RAIM) which is well-known in the field of GPS navigation. The result shows that the proposed method has better performance.

Table 1. Result of Wilcoxon Score Function.

SV No.	2	5	9	11	14	19	22	25	28
Score	9	7	6	4	1	2	8	3	5

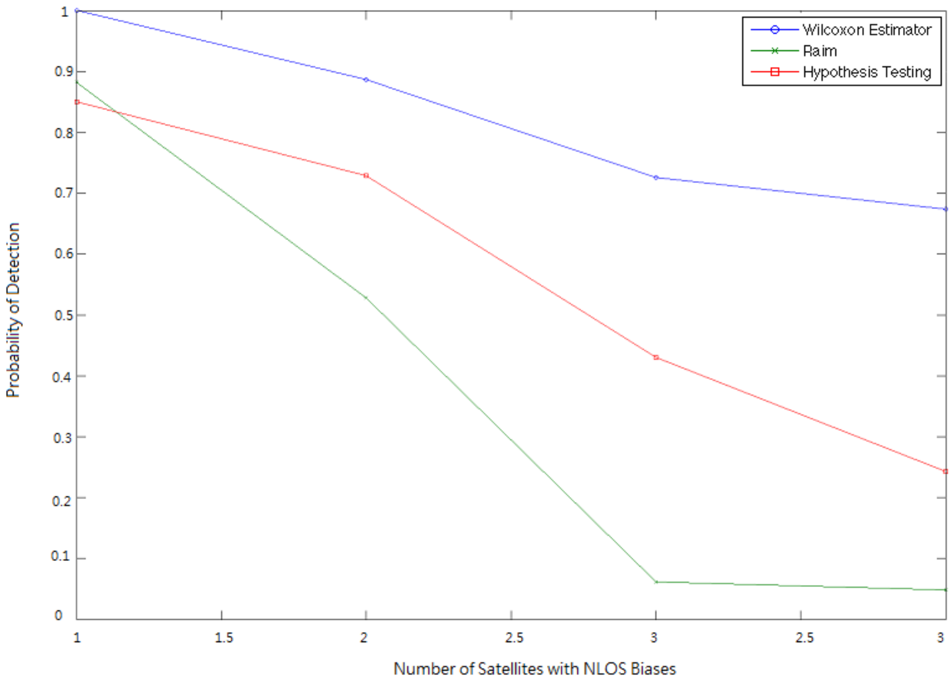


Figure 1. Performance Comparison of Three Methods.

Copyright © 2013 International Journal of Automation and Smart Technology

Another Monte Carlo simulation is conducted to evaluate the performance of the NLOS identification method described in the previous section. In the present simulation, an artificial multipath profile is superimposed on a set of real collected GPS pseudorange measurements. A few iterative optimization-based algorithms are selected for performance evaluation. Those methods are taken from [10] and [1], and are not presented in detail here. The traditional LS (least-squares) algorithm is also used for comparison. Note that all the methods are originally proposed for multipath mitigation or NLOS detection in the field of non-GPS-based wireless positioning; therefore some minor modifications must be made before applying those methods to GPS positioning problems. To compare the methods, a Monte Carlo simulation is run for a scenario with ten in-view satellites. To evaluate the bias mitigation performance, NLOS errors are purposely added to the pseudorange measurements. The NLOS error is assumed to have a Rayleigh distribution with a multipath envelope ranging from 5 to 200 meters. For GNSS navigation, the multipath error usually ranges from 5 (in a moderate environment) to 100 meters (in an urban canyon environment). The highly adverse values here are only used for the purposes of demonstration and may not reflect a real-world situation. Figure 2 shows the cumulative distribution functions (CDFs) of the location errors of the four different methods given six measurements with NLOS biases. The use of CDFs to evaluate the performance of multipath mitigation algorithms is suggested by [7] and [1]. Method 1 is the proposed algorithm. Methods 2 and 3 correspond to the algorithms described in [10] and [1], and are respectively denoted as “SQP-based method” (without the use of the Wilcoxon regression) and “TS-LQP method”. The fourth method is the traditional LS method. It can be easily seen that the proposed method outperforms the other methods.

The previous experiment shows that, although the proposed algorithm has better performance, the positioning error may be greater than 50 meters most of time (around 50% CDF). For the next experiment, we assume that the NLOS signals can be correctly identified.

In this case, we want to see whether the proposed algorithm is able to accurately estimate the bias errors. The results presented in Figures 3 and 4 show that the positioning error is greatly reduced if the satellite signals which are indeed contaminated by the multipath effect can be identified prior to applying the proposed method, thus allowing us to impose more specific constraints on the nonlinear programming problem.

The final simulation examines the dynamic performance of the proposed method. We assume that the NLOS bias is a Gaussian distribution with a mean equal to 100 meters and a variance of 20 meters. This is only for demonstration purposes, since the behavior of multipath errors is usually rather smooth and tends to average out over time. The simulation results shown in Figure 5 indicates that the proposed method is able to accurately estimate the NLOS biases.

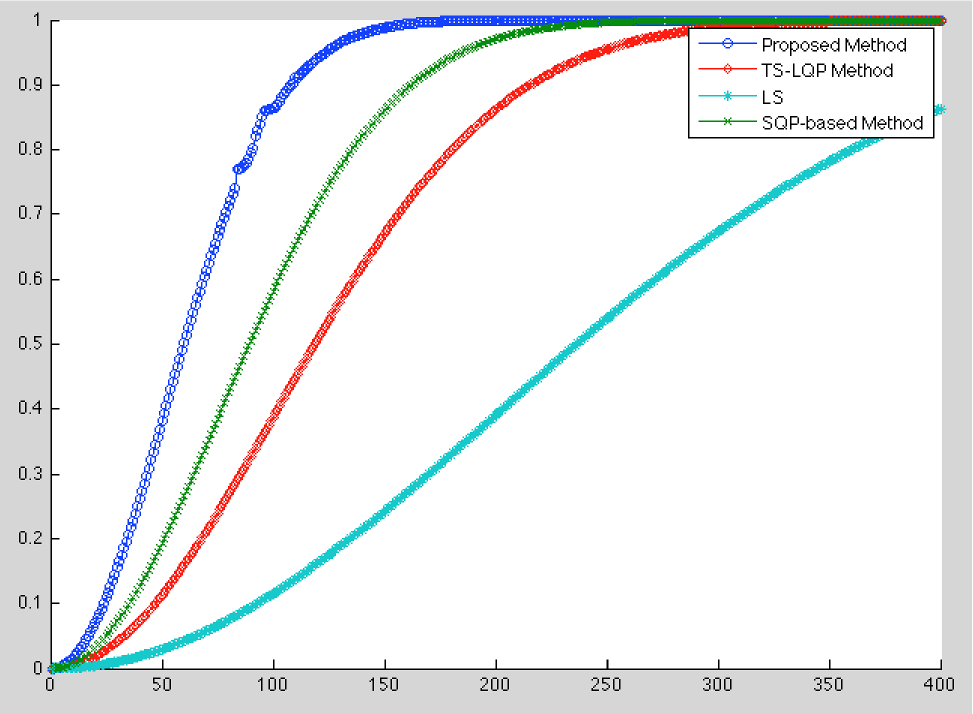
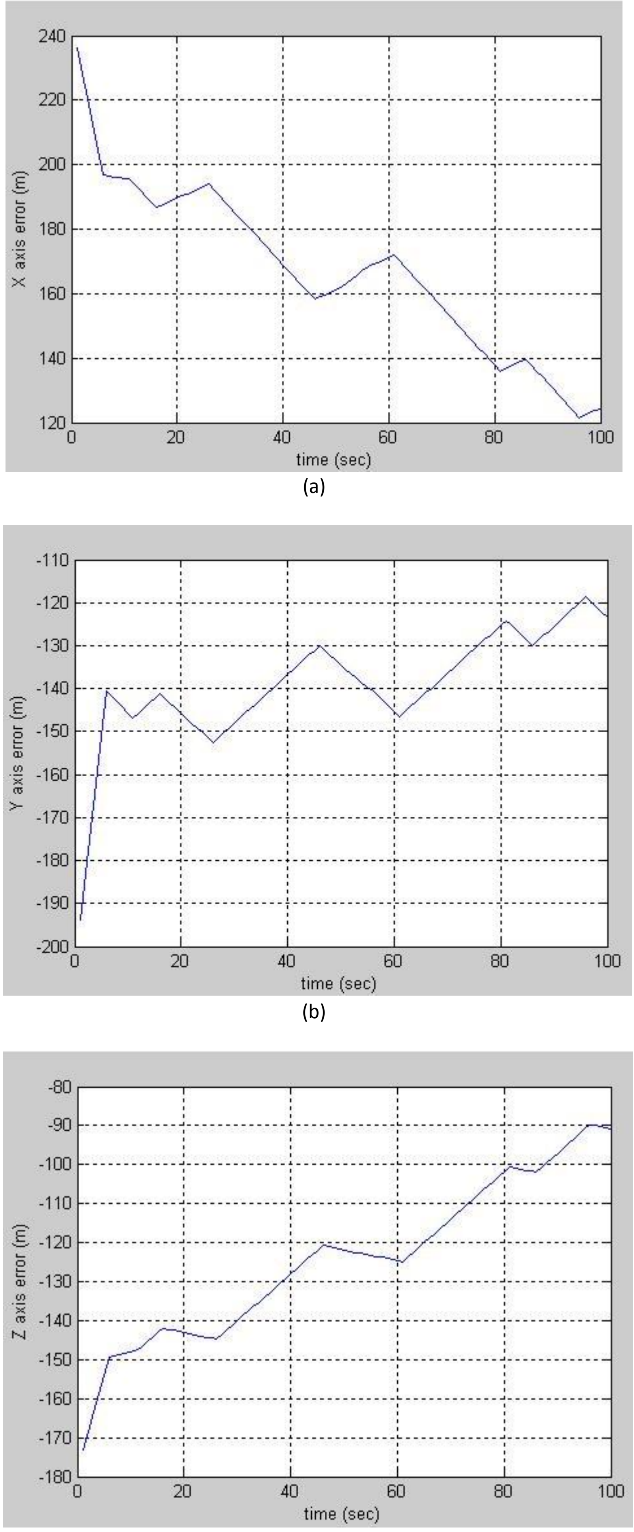


Figure 2. Positioning error CDPs of four different NLOS detection algorithms for six NLOS measurements among ten satellite signals.

Copyright © 2013 International Journal of Automation and Smart Technology

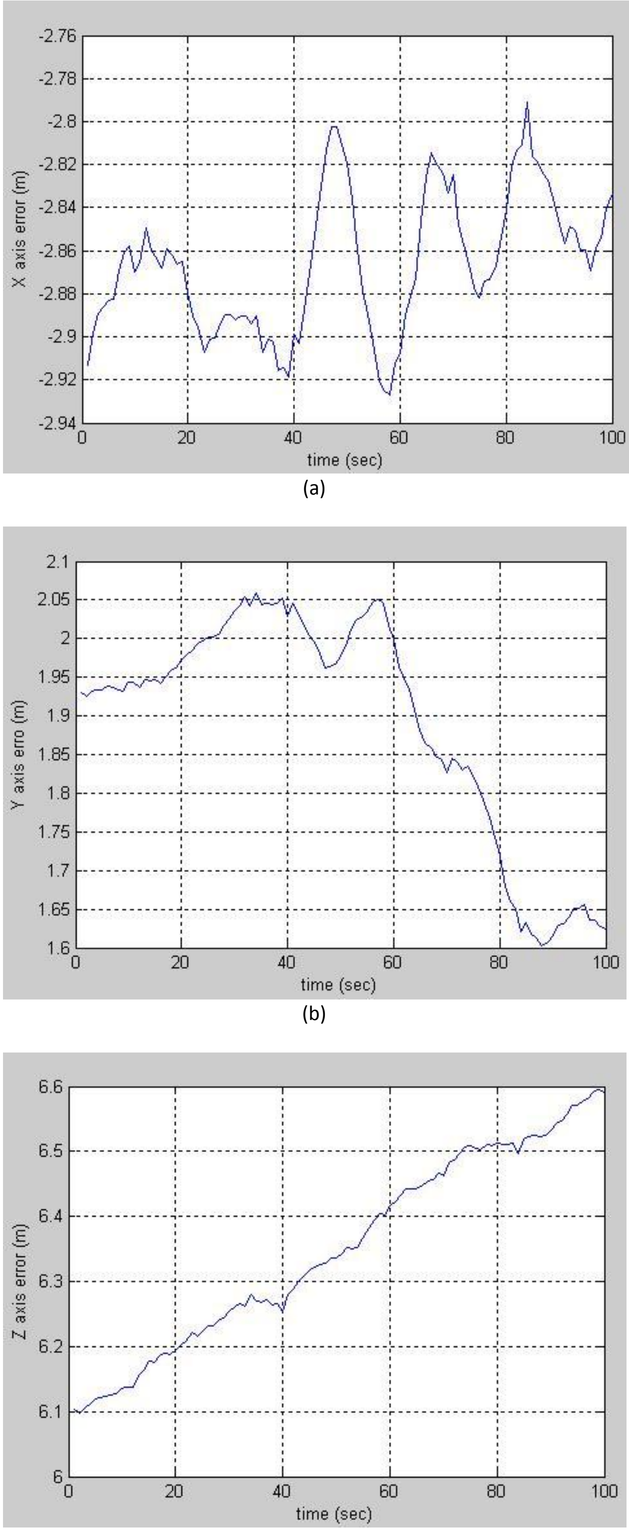


(a)

(b)

(c)

Figure 3. (a), (b), and (c) respectively show the *x*, *y*, *z*-axes errors prior to NLOS elimination.



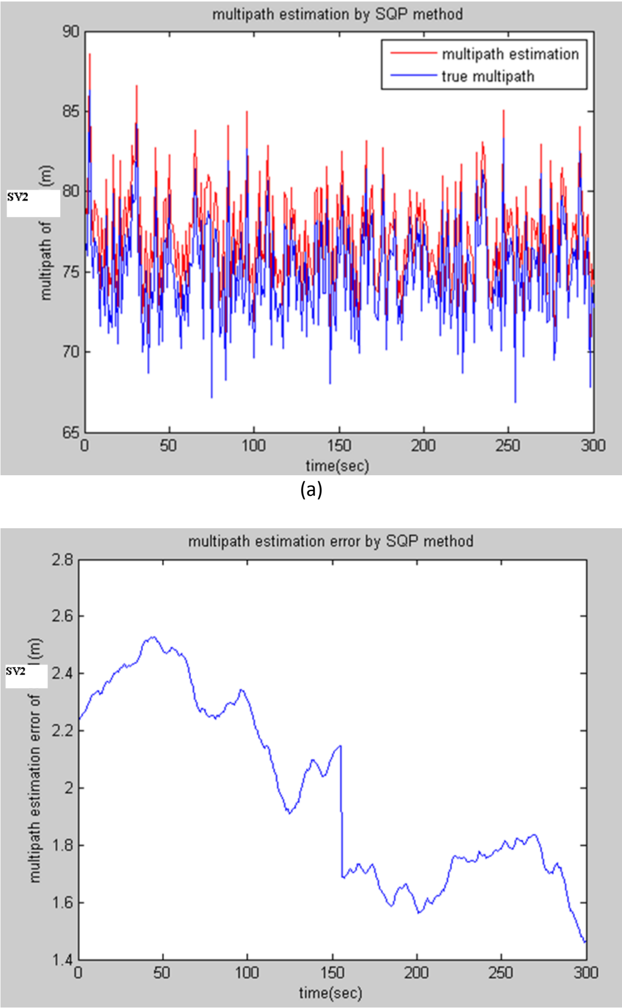
(a)

(b)

(c)

Figure 4. (a), (b), and (c) respectively show the *x, y, z*-axes errors after the NLOS biases are removed by the proposed algorithm.

Copyright © 2013 International Journal of Automation and Smart Technology



(a)

(b)

Figure 5. NLOS Bias Error Estimation (assume NLOS bias is Gaussian distributed with mean = 100 meters, and variance = 20 meters).

5. Conclusion

Recent advances in wireless positioning technology have offered considerable improvement in NLOS mitigation techniques. This paper applies some of the new methods developed in this field to GPS multipath error mitigation. The present work is the first to use SQP optimization to solve GPS NLOS detection and mitigation problems. Experimental results show that the proposed algorithm is able to accurately estimate the bias error. We also present a robust filter-based method for identifying satellite signals that contain NLOS transmission prior to applying the constraint optimization technique. By doing so, the number of constrained equations can be reduced thus substantially improving the estimation result.

6. Appendix

Detailed calculations of the matrices $\mathbf{A}_k$ and $\mathbf{B}_k$ are shown. The matrix $\mathbf{A}_k$ = [$\mathbf{A}(1)$ $\mathbf{A}(2)$ … $\mathbf{A}(n+4)$], where

$$\mathbf{A}(1)=\frac{\partial f}{\partial x}=2\sum_{i=1}^{n}\left(\boldsymbol{\rho}(i)-\left\|\mathbf{u}-\mathbf{s}^{i}\right\|-b-\mathbf{m}(i)\right)\left(-\frac{x-x^{i}}{\left\|\mathbf{u}-\mathbf{s}^{i}\right\|}\right),$$

$$\mathbf{A}(2)=\frac{\partial f}{\partial y}=2\sum_{i=1}^{n}\left(\boldsymbol{\rho}(i)-\left\|\mathbf{u}-\mathbf{s}^{i}\right\|-b-\mathbf{m}(i)\right)\left(-\frac{y-y^{i}}{\left\|\mathbf{u}-\mathbf{s}^{i}\right\|}\right),$$

$$\mathbf{A}(3)=\frac{\partial f}{\partial z}=2\sum_{i=1}^{n}\left(\boldsymbol{\rho}(i)-\left\|\mathbf{u}-\mathbf{s}^{i}\right\|-b-\mathbf{m}(i)\right)\left(-\frac{z-z^{i}}{\left\|\mathbf{u}-\mathbf{s}^{i}\right\|}\right),$$

$$\mathbf{A}(4)=\frac{\partial f}{\partial b}=2\sum_{i=1}^{n}\left(\boldsymbol{\rho}(i)-\left\|\mathbf{u}-\mathbf{s}^{i}\right\|-b-\mathbf{m}(i)\right)(-1),$$

$$\begin{aligned}\mathbf{A}(i)&=\frac{\partial f}{\partial \mathbf{m}(i-4)}\\&=2\left(\boldsymbol{\rho}(i-4)-\left\|\mathbf{u}-\mathbf{s}^{i-4}\right\|-b-\mathbf{m}(i-4)\right)(-1);i=5\cdots n+4.\end{aligned}$$

$\mathbf{B}$ = [$\mathbf{B}(i,j)$] is an $(n+4)$-by-$(n+4)$ matrix, where

$$\begin{aligned}\mathbf{B}(1,1)&=\frac{\partial^{2} f}{\partial x^{2}}\\&=2\sum_{i=1}^{n}\left\{\begin{pmatrix}\boldsymbol{\rho}(i)-\left\|\mathbf{u}-\mathbf{s}^{i}\right\|\\-b-\mathbf{m}(i)\end{pmatrix}\left[\left(-\frac{x-x^{i}}{\left\|\mathbf{u}-\mathbf{s}^{i}\right\|}\right)^{2}+\frac{-\left\|\mathbf{u}-\mathbf{s}^{i}\right\|+x\dfrac{x-x^{i}}{\left\|\mathbf{u}-\mathbf{s}^{i}\right\|}}{\left\|\mathbf{u}-\mathbf{s}^{i}\right\|^{2}}\right]\right\}\end{aligned}$$

$$\begin{aligned}\mathbf{B}(2,2)&=\frac{\partial^{2} f}{\partial y^{2}}\\&=2\sum_{i=1}^{n}\left\{\begin{pmatrix}\boldsymbol{\rho}(i)-\left\|\mathbf{u}-\mathbf{s}^{i}\right\|\\-b-\mathbf{m}(i)\end{pmatrix}\left[\left(-\frac{y-y^{i}}{\left\|\mathbf{u}-\mathbf{s}^{i}\right\|}\right)^{2}+\frac{-\left\|\mathbf{u}-\mathbf{s}^{i}\right\|+y\dfrac{y-y^{i}}{\left\|\mathbf{u}-\mathbf{s}^{i}\right\|}}{\left\|\mathbf{u}-\mathbf{s}^{i}\right\|^{2}}\right]\right\}\end{aligned}$$

$$\begin{aligned}\mathbf{B}(3,3)&=\frac{\partial^{2} f}{\partial z^{2}}\\&=2\sum_{i=1}^{n}\left\{\begin{pmatrix}\boldsymbol{\rho}(i)-\left\|\mathbf{u}-\mathbf{s}^{i}\right\|\\-b-\mathbf{m}(i)\end{pmatrix}\left[\left(-\frac{z-z^{i}}{\left\|\mathbf{u}-\mathbf{s}^{i}\right\|}\right)^{2}+\frac{-\left\|\mathbf{u}-\mathbf{s}^{i}\right\|+z\dfrac{z-z^{i}}{\left\|\mathbf{u}-\mathbf{s}^{i}\right\|}}{\left\|\mathbf{u}-\mathbf{s}^{i}\right\|^{2}}\right]\right\}\end{aligned}$$

Copyright © 2013 International Journal of Automation and Smart Technology

$$\mathbf{B}(1,2)=\frac{\partial^2 f}{\partial xy}=\mathbf{B}(2,1)=\frac{\partial^2 f}{\partial yx}$$

$$=2\sum_{i=1}^{n}\left\{\left(\begin{matrix}\boldsymbol{\rho}(i)-\left\|\mathbf{u}-\mathbf{s}^i\right\|\\-b-\mathbf{m}(i)\end{matrix}\right)\left[\left(-\frac{x-x^i}{\left\|\mathbf{u}-\mathbf{s}^i\right\|}\right)\left(-\frac{y-y^i}{\left\|\mathbf{u}-\mathbf{s}^i\right\|}\right)\right]\right\}$$

$$\mathbf{B}(1,3)=\frac{\partial^2 f}{\partial xz}=\mathbf{B}(3,1)=\frac{\partial^2 f}{\partial zx}$$

$$=2\sum_{i=1}^{n}\left\{\left(\begin{matrix}\boldsymbol{\rho}(i)-\left\|\mathbf{u}-\mathbf{s}^i\right\|\\-b-\mathbf{m}(i)\end{matrix}\right)\left[\left(-\frac{x-x^i}{\left\|\mathbf{u}-\mathbf{s}^i\right\|}\right)\left(-\frac{z-z^i}{\left\|\mathbf{u}-\mathbf{s}^i\right\|}\right)\right]\right\}$$

$$\mathbf{B}(2,3)=\frac{\partial^2 f}{\partial yz}=\mathbf{B}(3,2)=\frac{\partial^2 f}{\partial zy}$$

$$=2\sum_{i=1}^{n}\left\{\left(\begin{matrix}\boldsymbol{\rho}(i)-\left\|\mathbf{u}-\mathbf{s}^i\right\|\\-b-\mathbf{m}(i)\end{matrix}\right)\left[\left(-\frac{y-y^i}{\left\|\mathbf{u}-\mathbf{s}^i\right\|}\right)\left(-\frac{z-z^i}{\left\|\mathbf{u}-\mathbf{s}^i\right\|}\right)\right]\right\}$$

$$\mathbf{B}(4,4)=\frac{\partial^2 f}{\partial b^2}=2n$$

$$\mathbf{B}(1,4)=\frac{\partial^2 f}{\partial xb}=\mathbf{B}(4,1)=\frac{\partial^2 f}{\partial bx}=2\sum_{i=1}^{n}\left(\frac{x-x^i}{\left\|\mathbf{u}-\mathbf{s}^i\right\|}\right)$$

$$\mathbf{B}(2,4)=\frac{\partial^2 f}{\partial yb}=\mathbf{B}(4,2)=\frac{\partial^2 f}{\partial by}=2\sum_{i=1}^{n}\left(\frac{y-y^i}{\left\|\mathbf{u}-\mathbf{s}^i\right\|}\right)$$

$$\mathbf{B}(3,4)=\frac{\partial^2 f}{\partial zb}=\mathbf{B}(4,3)=\frac{\partial^2 f}{\partial bz}=2\sum_{i=1}^{n}\left(\frac{z-z^i}{\left\|\mathbf{u}-\mathbf{s}^i\right\|}\right)$$

$$\mathbf{B}(i,i)=2; i=5\cdots n+4$$

$$\mathbf{B}(1,i+4)=\frac{\partial^2 f}{\partial x\mathbf{m}(i)}=\mathbf{B}(i+4,1)=\frac{\partial^2 f}{\partial \mathbf{m}(i)x}=2\frac{x-x^i}{\left\|\mathbf{u}-\mathbf{s}^i\right\|};$$

$i=1\cdots n.$

$$\mathbf{B}(2,i+4)=\frac{\partial^2 f}{\partial y\mathbf{m}(i)}=\mathbf{B}(i+4,2)=\frac{\partial^2 f}{\partial \mathbf{m}(i)y}=2\frac{y-y^i}{\left\|\mathbf{u}-\mathbf{s}^i\right\|};$$

$i=1\cdots n.$

$$\mathbf{B}(3,i+4)=\frac{\partial^2 f}{\partial z\mathbf{m}(i)}=\mathbf{B}(i+4,3)=\frac{\partial^2 f}{\partial \mathbf{m}(i)z}=2\frac{z-z^i}{\left\|\mathbf{u}-\mathbf{s}^i\right\|};$$

$i=1\cdots n.$

$$\mathbf{B}(4,i+4)=\frac{\partial^2 f}{\partial b\mathbf{m}(i)}=\mathbf{B}(i+4,4)=\frac{\partial^2 f}{\partial \mathbf{m}(i)b}=2;$$

$i=1\cdots n.$

$$\mathbf{B}(i,j)=\frac{\partial^2 f}{\partial \mathbf{m}(i-4)\mathbf{m}(j-4)}=\mathbf{B}(j,i)=\frac{\partial^2 f}{\partial \mathbf{m}(j-4)\mathbf{m}(i-4)}=0;$$

$i=5\cdots 5+(n-2), j=i+1\cdots n+4.$

Acknowledgement

The work was partially supported by the National Science Council, Taiwan, under Grant NSC 98-2221-E-019 -020-MY3.

References

[1] K. Yu and Y. J. Guo, "Statistical NLOS identification based on AOA, TOA, and signal strength," *IEEE Transactions on Vehicular Technology,* vol. 58, no. 1, pp. 274-286, 2009.
doi: 10.1109/TVT.2008.924975

[2] P. C. Chen, "Mobile position location estimation in cellular systems," Ph.D. Dissertation, Electrical and Computer Engineering Department, Rutgers University, New Jersey, US, 1999.

[3] H. S. Wang and W. C. Yang, "GBAS testbed development in taiwan with a prototype GPS/GBAS receiver," *GPS Solutions,* vol. 10, no. 3, pp. 197-206, 2006/07/01, 2006.
doi: 10.1007/s10291-006-0021-0

[4] S. Juurakko and W. Backman, "Database correlation method with error correction for emergency location," *Wireless Personal Communications,* vol. 30, no. 2-4, pp. 183-194, 2004/09/01, 2004.
doi: 10.1023/B:WIRE.0000049398.10614.02

[5] L. Cong and W. Zhuang, "Nonline-of-sight error mitigation in mobile location," in *The Twenty-third AnnualJoint Conference of the IEEE Computer and Communications Societies (INFOCOM)*, 2004, p. 659.
doi: 10.1109/INFCOM.2004.1354536

[6] K. Yu, I. Sharp, and Y. J. Guo, *Ground-based wireless positioning*. Chichester, UK: Wiley, 2009.

[7] K. Wuk, J. G. Lee, and G. I. Jee, "The interior-point method for an optimal treatment of bias in trilateration location," *IEEE Transactions on Vehicular Technology,* vol. 55, no. 4, pp. 1291-1301, 2006.
doi: 10.1109/TVT.2006.877760

[8] R. B. Wilson, "A simplicial method for convex programming," Ph.D. Dissertation, Graduate School of Business Administration, Harvard University, Cambridge, MA, 1963.

[9] M. A. Sturza, "Navigation system integrity monitoring using redundant measurements," *Journal of the Institute of Navigation,* vol. 35, no. 4, pp. 483-502, 1988.
Available: http://www.ion.org/search/view_abstract.cfm?jp=j&idno=343

[10] J. Jr. Caffery and G. L. Stuber, "Subscriber location in CDMA cellular networks," *IEEE Transactions on Vehicular Technology,* vol. 47, no. 2, pp. 406-416, 1998.
doi: 10.1109/25.669079

[11] S. P. Boyd and L. Vandenberghe, *Convex optimization*. Cambridge, UK; New York: Cambridge University Press, 2004.

[12] Y. Bar-Shalom and X. R. Li, *Estimation and tracking : Principles, techniques, and software*. Boston: Artech House, 1993.

Copyright © 2013 International Journal of Automation and Smart Technology

[13] S. M. Kay, *Fundamentals of statistical signal processing 2, detection theory*. Upper Saddle River, NJ: Prentice Hall PTR, 1998.

[14] D. G. Luenberger and Y. Ye, *Linear and nonlinear programming*, 3 ed. New York, NY: Springer, 2008.

[15] S. Venkatraman, J. Jr. Caffery, and H. R. You, "Location using LOS range estimation in NLOS environments," in *IEEE 55th Vehicular Technology Conference*, 2002, pp. 856-860.
doi: 10.1109/VTC.2002.1002609

[16] F. van Diggelen and A. Brown, "Mathematical aspects of GPS RAIM," in *IEEE Position Location and Navigation Symposium*, Las Vegas, NV, 1994, pp. 733-738.
doi: 10.1109/PLANS.1994.303383

[17] S. Al-Jazzar and J. Caffery, Jr., "NLOS mitigation method for urban environments," in *IEEE 60th Vehicular Technology Conference (VTC)*, 2004, pp. 5112-5115 Vol. 7.
doi: 10.1109/VETECF.2004.1405073

[18] M. P. Wylie and J. Holtzman, "The non-line of sight problem in mobile location estimation," in *The 5th IEEE International Conference on Universal Personal Communications*, 1996, pp. 827-831.
doi: 10.1109/ICUPC.1996.562692

[19] W. Xin, W. Zongxin, and B. O'Dea, "A TOA-based location algorithm reducing the errors due to non-line-of-sight (NLOS) propagation," *IEEE Transactions on Vehicular Technology,* vol. 52, no. 1, pp. 112-116, 2003.
doi: 10.1109/TVT.2002.807158

Copyright © 2013 International Journal of Automation and Smart Technology