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Assessing Measurement Noise Effect in Run-to-Run Process Control: Extends EWMA Controller by Kalman Filter

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Abstract: Recently, the Exponentially Weighted Moving Average (EWMA) controller has become a popular control method in Run-to-Run (RtR) process control, but the issue of measurement noise from metrology tools has not been addressed in RtR EWMA controllers yet. This paper utilizes a Kalman Filter (KF) controller to deal with measurement noise in RtR process control and investigates the output properties for steady-state mean and variance, and for closed-loop stability. Five disturbance models modeling semiconductor process disturbances are investigated. These disturbance models consist of Deterministic Trend (DT), Random Walk with Drift (RWD), Integrated Moving Average process (IMA(1,1)), AutoRegressive Moving Average (ARMA(1,1)), and Autoregressive Integrated Moving Average (ARIMA(1,1,1)). Analytical results show that a KF controller can be considered as an extended version of a RtR EWMA controller. In particular, the EWMA controller is a special case of KF in a filtering form without the capability of measuring noise. Simulation results also show that the KF has a better ability to deal with measurement noise than the EWMA controller.

Keywords: Run-to-Run; Exponentially Weighted Moving Average (EWMA); Kalman Filter; measurement noise.

Introduction

Advanced process control (APC) has been recognized as a proper tool for improving semiconductor manufacturing efficiency and product quality. Currently, run-to-run (RtR) process control methods which perform reliably for APC applications are the most commonly used.

Spanos *et al.* [1] described a statistical process control (SPC) scheme that took advantage of real-time information and was capable of generating malfunction alarms but was not capable of prescribing control action. Sachs *et al.* [2] proposed a modular framework for implementing process control to the Low Pressure Chemical Vapor Deposition (LPCVD) of polysilicon in Very-large-Scale Integration (VLSI) fabrication. The system integrated existing approaches with new methodologies to achieve on-line optimization and control of unit process. Sachs *et al.* [3] provided a framework that combined SPC and feedback control for controlling processes affected by disturbances such as shifts and drifts. Additionally, the RtR controller was implemented and tested by application through the application of a silicon epitaxy process in a barrel reactor. In this research project the process was assumed to have no dynamics and a linear controller based on the exponentially weighted moving average (EWMA) was in used. The performance of this approach is highly dependent on the choice of EWMA controller parameters (EWMA weights), and the ability to dynamically update the EWMA weight. Some authors [4-8] have addressed this problem in their studies by proposing a self-tuning EWMA controller which dynamically updates its controller parameters. Performance and stability analyses of single-input/single-output (SISO) and multi-input/multi-output (MIMO) EWMA controllers in RtR process control have also been proposed by other authors [9-16]. In addition the high-mixed production issues in the control of semiconductor manufacturing processes that use the EWMA controller and its extension have been discussed by other authors [17-19]. Recently, Lee *et al*. [20] presented a unified framework called the Output Disturbance OBserver (ODOB) structure for the EWMA controller, the double EWMA controller and the PCC controller. The work enhances insight into well-known established algorithms, and contributes to a better understanding of how these algorithms operate and why they can be used successfully in practical applications. Other methods for RtR controllers have also been proposed and applied in semiconductor manufacturing [21, 22]. As can be seen in the above references, although both SISO and MIMO EWMA have been applied as successful RtR process control methods for APC application, the effect of the measurement noise coming from metrology tools has never been addressed. This omission is significant because the system output (yield rate) of semiconductor manufacturing could be badly impacted by measurement noise, especially in lithographic processes. Manufacturing companies set up multiple metrology tools to improve throughput problems; however, this arrangement admits different disturbance characteristics among the metrology tools. The RtR control method will perform better if the variation which is caused by each metrology tool is considered. In this paper, the proposal is made to extend EWMA controllers by linking them to Kalman filter (KF) controllers to deal with the measurement noise effect coming from one single metrology tool.

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This paper will first review the EWMA controller and a discrete KF. For different disturbances, state space equations are constructed corresponding to each disturbance to estimate disturbance states by using a KF. Next the paper examines system output properties when it is assumed that the EWMA-controlled process produces no measurement noise, and that the KF-controlled process produces measurement noise.. Finally, two simulation studies are presented to demonstrate output performances for both controllers in terms of asymptotic mean square deviation (AMSD) when the process produces metrology noise.

EWMA and Kalman Filter Controller

In general, the linear model for the semiconductor manufacturing process is usually assumed by the following algebraic equation.

$$y_k = \alpha + \beta u_k + \delta_k \tag{1}$$

where k denotes the batch number, y_k is the system measurement, u_k is the system recipe, α is the system intercept, β is the system gain, and δ_k is the process disturbance with noise. A few commonly found disturbance models in RtR processes control are listed below [13] and [23].

1. Deterministic Trend (DT)

$$\delta_k = \Delta k + \varepsilon_k = \delta_{k-1} + \Delta + \varepsilon_k - \varepsilon_{k-1}$$

The above equation can be represented in a state space form as:

$$\begin{bmatrix} \delta_k \\ \delta_{k-1} \\ \Delta_k \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \delta_{k-1} \\ \delta_{k-2} \\ \Delta_{k-1} \end{bmatrix} + \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} \varepsilon_k$$

$$\delta_k = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \delta_k \\ \delta_{k-1} \\ \Delta_k \end{bmatrix} \tag{2}$$

2. Random Walk with Drift (RWD)

$$\delta_k = \delta_{k-1} + \Delta + \varepsilon_k$$

Its state space form is represented as:

$$\begin{bmatrix} \delta_k \\ \Delta_k \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \delta_{k-1} \\ \Delta_{k-1} \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \varepsilon_k$$

$$\delta_k = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \delta_k \\ \Delta_k \end{bmatrix} \tag{3}$$

3. Integrated Moving Average Process (IMA(1,1))

$$\delta_k = \delta_{k-1} + \varepsilon_k - \theta\varepsilon_{k-1}$$

Its state space form is represented as:

$$\begin{bmatrix} \delta_k \\ \delta_{k-1} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta_{k-1} \\ \delta_{k-2} \end{bmatrix} + \begin{bmatrix} 1 \\ -\theta \end{bmatrix} \varepsilon_k$$

$$\delta_k = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \delta_k \\ \delta_{k-1} \end{bmatrix} \tag{4}$$

4. Autoregressive Moving Average Process (ARMA(1,1))

$$\delta_k = \phi\delta_{k-1} + \varepsilon_k - \theta\varepsilon_{k-1}$$

Its state space form is represented as:

$$\begin{bmatrix} \delta_k \\ \phi\delta_{k-1} \end{bmatrix} = \begin{bmatrix} \phi & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta_{k-1} \\ \phi\delta_{k-2} \end{bmatrix} + \begin{bmatrix} 1 \\ -\theta \end{bmatrix} \varepsilon_k$$

$$\delta_k = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \delta_k \\ \phi\delta_{k-1} \end{bmatrix} \tag{5}$$

5. Autoregressive Integrated Moving Average Process (ARIMA(1,1,1))

$$\delta_k = (1+\phi)\delta_{k-1} - \phi\delta_{k-2} + \varepsilon_k - \theta\varepsilon_{k-1}$$

Its state space form is represented as:

$$\begin{bmatrix} \delta_k \\ \phi\delta_{k-1} \end{bmatrix} = \begin{bmatrix} 1+\phi & 1 \\ -\phi & 0 \end{bmatrix} \begin{bmatrix} \delta_{k-1} \\ \phi\delta_{k-2} \end{bmatrix} + \begin{bmatrix} 1 \\ -\theta \end{bmatrix} \varepsilon_k$$

$$\delta_k = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \delta_k \\ \phi\delta_{k-1} \end{bmatrix} \tag{6}$$

In (2)-(6), ε_k denotes the white noise with zero mean and variance σ_ε ($\varepsilon_k \sim N(0, \sigma_\varepsilon^2)$), Δ is the per time unit expected drift; ϕ and θ denote the autoregressive and moving average coefficients respectively. Figure 1 shows one realization of each disturbance in (2)-(6) for the case when ϕ=θ=0.5, Δ=0.2 and $\varepsilon_k \sim N(0, 1)$. Note that, contrary to the DT disturbance, the RWD disturbance is not "cementing" to the line defined by a linear trend with slope Δ, but eventually it drifts in the same direction as given by the slope Δ. The IMA(1,1) and ARIMA(1,1,1) disturbances drift erratically, not "attaching" to the line defined by the time axis.

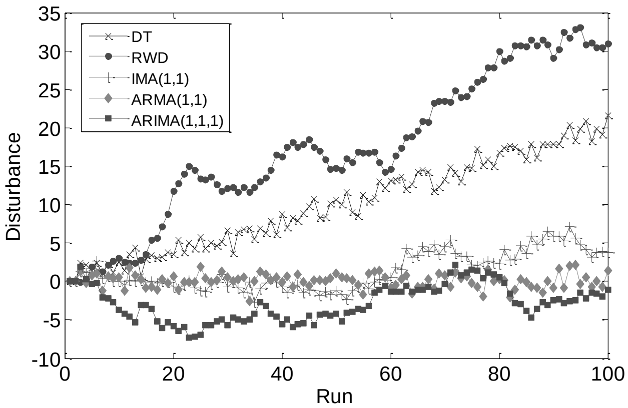


Figure 1. Realizations of each disturbance model.

The EWMA controller is summarized by the following equation:

$$\hat{d}_k = \lambda(y_k - bu_k - \alpha) + (1-\lambda)\hat{d}_{k-1} \tag{7}$$

where $\hat{d}_k$ denotes the model intercept estimation, and λ is the weight ($0 < \lambda < 1$).

A dead-beat controller is often used:

$$u_{k+1} = \frac{T - \hat{d}_k - \alpha}{b}, \tag{8}$$

where T is the target value of the quality characteristic, b is an off-line estimate of β usually determined using Design Of Experiments (DOE) and regression analysis.

In order to deal with measurement noise, the KF is utilized as an extension to the EWMA algorithm. Equation (1) can be represented in a state space form with added measurement noise as follows:

$$\boldsymbol{\delta}_{k+1} = \mathbf{A}\boldsymbol{\delta}_k + \mathbf{G}\varepsilon_{k+1} \tag{9}$$

$$y_k = \mathbf{C}\boldsymbol{\delta}_k + \beta u_k + \alpha + v_k \tag{10}$$

where v_k denotes the measurement noise ($v_k \sim N(0, \sigma_v^2)$), and **A**, **C**, **G** denote coefficient matrices and α denotes constant. In addition, the process noise, measurement noise and process state have the following properties:

$$E[\varepsilon_k \varepsilon_j^T] = Q\Delta_{kj} \quad Q \geq 0, \ \forall k, j \geq 0$$

$$E[v_k v_j^T] = R\Delta_{kj}, \quad R \geq 0, \ \forall k, j \geq 0$$

$$E[v_k \delta_k] = 0 \quad \forall k \geq 0$$

where Δ_{kj} stands for the Kronecker delta function. Thus, the $(k+1)^{th}$ state and state error covariance ($\hat{\boldsymbol{\delta}}_{k+1|k}$ and $\mathbf{P}_k$) can be predicted by the discrete KF [24, 25] algorithm.

$$\mathbf{P}_{k|k-1} = \mathbf{A}\mathbf{P}_{k-1}\mathbf{A}^T + \mathbf{G}Q\mathbf{G}^T \tag{11}$$

$$\mathbf{K}_k = \mathbf{P}_{k|k-1}\mathbf{C}^T \left(\mathbf{C}\mathbf{P}_{k|k-1}\mathbf{C}^T + R\right)^{-1} \tag{12}$$

$$\hat{\boldsymbol{\delta}}_{k|k} = \hat{\boldsymbol{\delta}}_{k|k-1} + \mathbf{K}_k \left(y_k - \mathbf{C}\hat{\boldsymbol{\delta}}_{k|k-1} - bu_k - \alpha\right) \tag{13}$$

$$\mathbf{P}_k = \left(\mathbf{I} - \mathbf{K}_k\mathbf{C}\right)\mathbf{P}_{k|k-1} \tag{14}$$

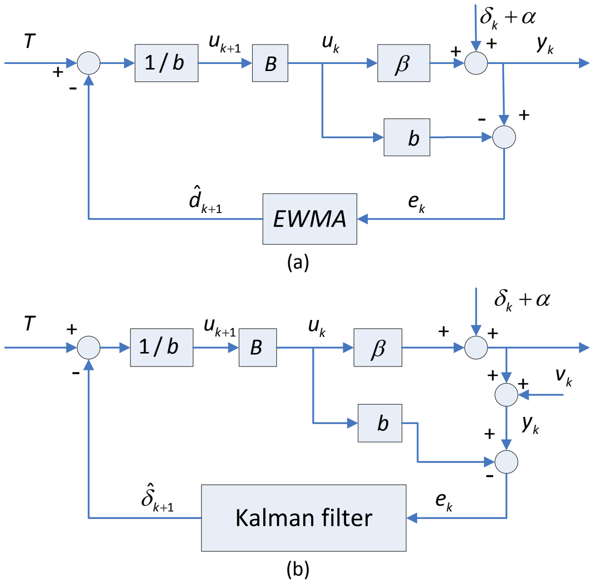
$$\hat{\boldsymbol{\delta}}_{k+1|k} = \mathbf{A}\hat{\boldsymbol{\delta}}_{k|k} \tag{15}$$

The next batch's recipes can be generated by the following equations.

$$u_{k+1} = \frac{T - \mathbf{C}\hat{\boldsymbol{\delta}}_{k+1|k} - \alpha}{b} \tag{16}$$

$$u_{k+1} = \frac{T - \mathbf{C}\hat{\boldsymbol{\delta}}_{k|k} - \alpha}{b} \tag{17}$$

In equation (16), one-step-ahead prediction of $\hat{\boldsymbol{\delta}}_{k|k}$, $\hat{\boldsymbol{\delta}}_{k+1|k}$, by equation (15) is used to generate the next recipe, while in equation (17) the current estimate of $\hat{\boldsymbol{\delta}}_{k|k}$ is in use; the latter is what the EWMA controller uses in equation (8). Figure 2 presents the block diagrams of EWMA controller (Figure 2(a)) and KF (Figure 2(b)), where B denotes backward operator, i.e., $Bu_{k+1} = u_k$.



(a)

(b)

Figure 2. The block diagrams of (a) EWMA controller (b) Kalman filter.

Properties of RtR Process Control with Measurement Noise

This section begins with a discussion of the properties of five types of disturbance produced by EWMA-controlled processes without measurement noise. The asymptotic mean and variance of the KF controller for disturbances with measurement noise will be presented, and the relationship between EWMA and KF will be discussed subsequently. Finally, we will discuss briefly the stability of EWMA and KF controllers for the closed-loop system.

Properties of the EWMA Controller without Measurement Noise

Several authors have analyzed the EWMA controller. It is worth mentioning that Ingolfsson *et al.* [11] and del Castillo [13] have shown the results of the asymptotic mean and variance when the disturbances are DT, RWD and IMA(1,1). Here we go further to consider more complicated cases of ARMA(1,1) and ARIMA(1,1,1) disturbances. The results of asymptotic mean and variance for various disturbances without measurement noise are summarized as follows.

1. DT [13]

$$\lim_{k\to\infty} E[y_k] = T + \frac{\Delta}{\xi\lambda} \tag{18}$$

$$\lim_{k\to\infty} \mathrm{var}[y_k] = \frac{2}{2 - \lambda\xi}\sigma_\varepsilon^2 \tag{19}$$

2. RWD [13]

$$\lim_{k\to\infty} E[y_k] = T + \frac{\Delta}{\xi\lambda} \tag{20}$$

$$\lim_{k\to\infty} \mathrm{var}[y_k] = \frac{1}{\lambda\xi(2 - \lambda\xi)}\sigma_\varepsilon^2 \tag{21}$$

3. IMA(1,1) [13]

$$\lim_{k\to\infty} E[y_k] = T \tag{22}$$

$$\lim_{k\to\infty} \mathrm{var}[y_k] = \frac{[1 - 2(1 - \lambda\xi)\theta + \theta^2]}{\lambda\xi(2 - \lambda\xi)}\sigma_\varepsilon^2 \tag{23}$$

4. ARMA(1,1)

$$\lim_{k\to\infty} E[y_k] = T \tag{24}$$

$$\lim_{k\to\infty}\operatorname{var}[y_k] = \frac{2\{1+\theta[\theta-2\phi+\lambda\xi(1+\phi)]\}}{(2-\lambda\xi)(1+\phi)[1-(1-\lambda\xi)\phi]}\sigma_\varepsilon^2 \tag{25}$$

5. ARIMA(1,1,1)

$$\lim_{k\to\infty} E[y_k] = T \tag{26}$$

$$\lim_{k\to\infty}\operatorname{var}[y_k] = \frac{(1+\phi-\lambda\xi\phi)^2(\theta^2+1)-2\theta(1+\phi-\lambda\xi)}{\lambda\xi(2-\lambda\xi)(1-\phi^2)[1-(1-\lambda\xi)\phi]}\sigma_\varepsilon^2 \tag{27}$$

where $\xi = \beta / b$ is a measure of the mismatch or bias in the estimate of the gain. The EWMA controller can keep the steady-state system output on target except for DT and RWD disturbances. For more general disturbances in RtR process control (ARMA(1,1) and ARIMA(1,1,1)), the asymptotic mean of EWMA controller can reach the target, and the derivation details of ARMA(1,1) and ARIMA(1,1,1) disturbances are shown in Appendix A. Based on the results of IMA(1,1), ARMA(1,1) and ARIMA(1,1,1), it is possible to see that the autoregressive and moving average coefficients affect the asymptotic variance magnitude. The equations (25) and (27) will be reduced to equation (23) if one sets $\phi = 1$ and $\phi = 0$ respectively.

Properties of the KF Controller with Measurement Noise

In the previous section, the asymptotic mean and variance analysis of EWMA controllers without measurement noise with different disturbances were examined. Considering the measurement noise coming from a single metrology tool in the semiconductor manufacturing processes, we extended the EWMA controller by the KF controller to analyze the asymptotic mean and variance of system output by matching two controllers. Specifically, equations (9), (10), (13) and (17) are used to derive the KF Algorithm. Notation commonly used in KF is changed to match the EWMA, that is, one uses $\hat{\boldsymbol{\delta}}_k$ and $\hat{\boldsymbol{\delta}}_{k-1}$ instead of $\hat{\boldsymbol{\delta}}_{k|k}$ and $\hat{\boldsymbol{\delta}}_{k|k-1}$ respectively in equation (13). Based on the assumption that the $\mathbf{P}_k$ is converged in the steady state, we apply constant Kalman gain ($\mathbf{K}$) in the following analysis, i.e., we take the steady-state Kalman gain ($\mathbf{K}$) to analyze the asymptotic mean and variance. Hence, with the restatement of equations (9) and (10), we have

$$\boldsymbol{\delta}_{k+1} = \mathbf{A}\boldsymbol{\delta}_k + \mathbf{G}\varepsilon_{k+1} \tag{9}$$

$$y_k = \mathbf{C}\boldsymbol{\delta}_k + \beta u_k + \alpha + v_k \tag{10}$$

$$\hat{\boldsymbol{\delta}}_k = \hat{\boldsymbol{\delta}}_{k-1} + \mathbf{K}\left(y_k - \mathbf{C}\hat{\boldsymbol{\delta}}_{k-1} - bu_k - \alpha\right) \tag{28}$$

$$u_{k+1} = \frac{T - \mathbf{C}\hat{\boldsymbol{\delta}}_k - \alpha}{b} \tag{29}$$

The results of the KF controller with measurement noise under (2)-(6) disturbances are summarized as follows. The derivation details are presented in Appendix B.

1. DT

$$\lim_{k\to\infty} E[y_k] = T + \frac{\Delta}{\xi K_1} \tag{30}$$

$$\lim_{k\to\infty}\operatorname{var}[y_k] = \frac{2}{2-K_1\xi}\sigma_\varepsilon^2 + \frac{2}{2-K_1\xi}\sigma_v^2 \tag{31}$$

2. RWD

$$\lim_{k\to\infty} E[y_k] = T + \frac{\Delta}{\xi K_1} \tag{32}$$

$$\lim_{k\to\infty}\operatorname{var}[y_k] = \frac{1}{K_1\xi(2-K_1\xi)}\sigma_\varepsilon^2 + \frac{2}{2-K_1\xi}\sigma_v^2 \tag{33}$$

3. IMA(1,1)

$$\lim_{k\to\infty} E[y_k] = T \tag{34}$$

$$\lim_{k\to\infty}\operatorname{var}[y_k] = \frac{[1-2(1-K_1\xi)\theta+\theta^2]}{K_1\xi(2-K_1\xi)}\sigma_\varepsilon^2 + \frac{2}{2-K_1\xi}\sigma_v^2 \tag{35}$$

4. ARMA(1,1)

$$\lim_{k\to\infty} E[y_k] = T \tag{36}$$

$$\lim_{k\to\infty}\operatorname{var}[y_k] = \frac{2\{1+\theta[\theta-2\phi+K_1\xi(1+\phi)]\}}{(2-K_1\xi)(1+\phi)[1-(1-K_1\xi)\phi]}\sigma_\varepsilon^2 + \frac{2}{2-K_1\xi}\sigma_v^2 \tag{37}$$

5. ARIMA(1,1,1)

$$\lim_{k\to\infty} E[y_k] = T \tag{38}$$

$$\begin{aligned}&\lim_{k\to\infty}\operatorname{var}[y_k]\\&=\frac{(1+\phi-K_1\xi\phi)(\theta^2+1)-2\theta(1+\phi-K_1\xi)}{K_1\xi(2-K_1\xi)(1-\phi^2)[1-(1-K_1\xi)\phi]}\sigma_\varepsilon^2\\&+\frac{2}{2-K_1\xi}\sigma_v^2\end{aligned}\qquad(39)$$

Note that if we set $\phi=1$ in ARMA(1,1) disturbance, equation (37) will be equal to equation (35).

From the above results, one observes that the asymptotic variance for each case contains two terms: one is related to σ_ε, caused by disturbance noise, and the other to σ_v, caused by measurement noise. The asymptotic means of the KF controller are the same as those of the EWMA controller, but the asymptotic variances of the KF controller are obviously larger than those of the EWMA controller. Furthermore, all of the terms with disturbance (σ_ε^2) are the same, if the Kalman gain in KF are set to the weight λ in EWMA. The reason that Kalman gain in KF and λ in EWMA play the same role can be observed from equations (7) and (28). Rewriting equation (7) and comparing it with equation (28), one obtains the following two equations.

$$\begin{aligned}\hat{d}_k&=\lambda(y_k-bu_k-\alpha)+(1-\lambda)\hat{d}_{k-1}\\&=\hat{d}_{k-1}+\lambda(y_k-\hat{d}_{k-1}-bu_k-\alpha)\end{aligned}\qquad(7)$$

$$\hat{\boldsymbol{\delta}}_k=\hat{\boldsymbol{\delta}}_{k-1}+\mathbf{K}\left(y_k-\mathbf{C}\hat{\boldsymbol{\delta}}_{k-1}-bu_k-\alpha\right).\qquad(28)$$

It is clear that the EWMA filter is a special case of KF, and the prediction state of EWMA ($\hat{d}_k$) is actually just a current state estimate ($\hat{\boldsymbol{\delta}}_{k|k}=\hat{\boldsymbol{\delta}}_k$) of KF. That means the EWMA only filters the output and input information up to k run, and does not offer the one-step-ahead prediction value for k+1 run, i.e., the prediction value of disturbance in the k+1 run is given by k run, i.e., $\hat{d}_{k+1|k}=\hat{d}_k$. In other words, EWMA is a special filtering form of KF without considering the measurement noise. Nevertheless, the KF (in the equations (15)) can predict the next run state value based on the observations and the dynamic characteristics of disturbances (the **A** matrix). If we use the prediction form to formulate KF, the results will be different from the EWMA. Since this paper is primarily concerned with the similarity and relationships between EWMA and KF, we will not discuss this issue further. There is one more thing to point out that since the EWMA filter cannot deal with disturbance and measurement noise separately, if the process contains process disturbance (IMA(1,1), $\delta_k=\delta_{k-1}+\varepsilon_k-\theta\varepsilon_{k-1}$) and measurement noise (v_k), the EWMA filter will process and predict the state by using new disturbance signals ($\delta'_k=\delta'_{k-1}+\varepsilon'_k-\theta'\varepsilon'_{k-1}$ where ε'_k is a combination of the ε_k and v_k).

Table 1. The stability regions of EWMA and KF controllers under RWD, DT, IMA(1,1), ARMA(1,1) and ARIMA(1,1,1) disturbances

	EWMA	*KF*
RWD	$\lvert 1-\lambda\xi\rvert<1$ [13]	$\lvert 1-K_1\xi\rvert<1$
Drift	$\lvert 1-\lambda\xi\rvert<1$ [13]	$\lvert 1-K_1\xi\rvert<1$
IMA(1,1)	$\lvert 1-\lambda\xi\rvert<1$ [13]	$\lvert 1-K_1\xi\rvert<1$
ARMA(1,1)	$\lvert 1-\lambda\xi\rvert<1$ and $\lvert\phi\rvert<1$	$\lvert 1-K_1\xi\rvert<1$ and $\lvert\phi\rvert<1$
ARIMA(1,1,1)	$\lvert 1-\lambda\xi\rvert<1$ and $\lvert\phi\rvert<1$	$\lvert 1-K_1\xi\rvert<1$ and $\lvert\phi\rvert<1$

Stability analysis

The stable region is analyzed based on the characteristic equation obtained from the system closed-loop transfer function. del Castillo [13] has shown the stability regions of the EWMA controller under DT, RWD and IMA(1,1) disturbances, i.e., the system will be stable if and only if $\lvert 1-\lambda\xi\rvert<1$. As for ARMA(1,1) and ARIMA(1,1,1) disturbances for EWMA controllers, one can obtain the result that the system will be stable if and only if $\lvert 1-\lambda\xi\rvert<1$ and $\lvert\phi\rvert<1$. In the KF case, the stability regions under those disturbances are the same as with EWMA. The results are summarized in Table 1.

Simulation

To compare the EWMA and KF controllers' performance with measurement noise, the simulation was performed by setting two model-mismatch cases: one is ξ=1 and the other is ξ=1.2. The optimal weight of the EWMA controller can be obtained in several ways. One is minimizing the asymptotic mean square deviation ($\mathrm{AMSD}\equiv\lim_{k\to\infty}E\left[(y_k-T)^2\right]$) with respect to λ [13], and the other is minimizing the mean square error (MSE) of system output. In the simulation, the method in [13] is adopted to obtain the optimal weight for EWMA and Kalman gain for KF. The derivation details for optimum Kalman gain are given in Appendix C. Other simulation parameters are set as follows: $\mathrm{Target}=0$, $\phi=0.5$, $\theta=0.1$, $\Delta=0.2$, $\varepsilon_k\sim N(0,\,1)$ and $v_k\sim N(0,\,1)$ respectively. All of the initial value of state ($\hat{d}_0$, $\hat{\boldsymbol{\delta}}_0$, $\mathbf{K}_0$, u_0 , y_0 and α) are zero, and $\mathbf{P}_0=\mathbf{I}$, Q=1 and R=1 for KF. First, we generate 1000 simulation data for the two control schemes under the five disturbances and calculate the AMSD of system output. Then we repeat the procedure for 100 runs and take the average of AMSD. In addition the KF with recursive Kalman gain by Eequations (11)-(16) is also applied for comparison. Note that in this case the closed-loop system will become a time-varying system. The simulation results are shown in

Tables 2 and 3.

Table 2. The simulation result of ξ=1 case

Disturbance type	*DT*	*RWD*	*IMA (1,1)*	*ARMA (1,1)*	*ARIMA (1,1,1)*
AMSD by EWMA	2.762	3.094	2.811	2.370	4.135
AMSD by Fixed KF	2.726	2.698	2.529	2.206	3.137
AMSD improvement over EWMA	1%	13%	10%	7%	24%
AMSD by recursive KF	2.035	2.619	2.529	2.082	3.101
AMSD improvement over EWMA	26%	15%	10%	12%	25%

Table 3. The simulation result of ξ=1.2 case

Disturbance type	*DT*	*RWD*	*IMA (1,1)*	*ARMA (1,1)*	*ARIMA (1,1,1)*
AMSD by EWMA	2.786	3.124	2.820	2.355	4.133
AMSD by Fixed KF	2.747	2.724	2.534	2.230	3.150
AMSD improvement over EWMA	1%	13%	10%	5%	24%
AMSD by recursive KF	2.053	2.686	2.570	2.106	3.213
AMSD improvement over EWMA	26%	14%	9%	11%	22%

Tables 2 and 3 demonstrate that the KF with fixed Kalman gain has better performance than the EWMA controller by an improvement on AMSD reduction of from 1% to 24%. Furthermore, in general, recursive Kalman gain performs better than the fixed Kalman gain. The use of recursive Kalman is more like a self-tuning EWMA controller and deserves further investigation.

Conclusion

In this paper, the KF controller is investigated and compared with the commonly-used EWMA controller in RtR process control when the process involves measurement noise from a single metrology tool. The analytical results show that the EWMA controller is a special case of KF in its filtering form when the process produces no measurement noise and the simulation results show that the KF has a greater capacity for dealing with process when measurement noise is present. The semiconductor manufacturing industry needs more robust controllers which can deal with disturbances and measurement noise. Further studies would be useful to investigate the prediction-form KF in the situations of mixed product processes and multiple metrology tools, to see if it can fulfill this need.

Appendix A

Derivation of asymptotic mean and variance of the EWMA controller.

For ARMA(1,1) disturbance

By substituting (8) into (1), one obtains system output as

$$y_k = \alpha + \xi\left(T - \hat{d}_{k-1} - \alpha\right) + \delta_k . \qquad \text{(A1)}$$

If the disturbance behaves like (5), then (7) becomes

$$\hat{d}_k = \lambda\left(\alpha + T\xi - \xi\alpha - T + \delta_k\right) + (1-\lambda\xi)\hat{d}_{k-1} . \qquad \text{(A2)}$$

Define the state vector $\mathbf{x}'_k = \left[\hat{d}_{k-1} \quad \delta_k\right]$, then one can built up the state-space representation

$$\mathbf{x}_{k+1} = \mathbf{A}\mathbf{x}_k + \mathbf{w}_k$$

$$y_k = \mathbf{C}\mathbf{x}_k + R \qquad \text{(A3)}$$

where

$$\mathbf{A} = \begin{bmatrix} 1-\lambda\xi & \lambda \\ 0 & \phi \end{bmatrix}, \quad \mathbf{w}_k = \begin{bmatrix} \lambda\left(\alpha - \xi\alpha + \xi T - T\right) \\ \varepsilon_{k+1} - \theta\varepsilon_k \end{bmatrix},$$

$$\mathbf{C} = \begin{bmatrix} -\xi & 1 \end{bmatrix} \text{ and } R = \alpha + \xi T .$$

The solution of $\mathbf{x}_k$ is given by

$$\mathbf{x}_k = \mathbf{A}^k\mathbf{x}_0 + \sum_{i=1}^{k-1}\mathbf{A}^{k-i-1}\mathbf{w}_i . \qquad \text{(A4)}$$

The matrix $\mathbf{A}^k$ is computed by diagonalization, namely $\mathbf{A}^k = \mathbf{P}\mathbf{\Gamma}^k\mathbf{P}^{-1}$ where $\mathbf{\Gamma}$ is diagonal matrix with eigenvalues of $\mathbf{A}$, and $\mathbf{P}$ is a matrix with the corresponding eigenvectors. It is not difficult to show that

$$\mathbf{\Gamma} = \begin{bmatrix} 1-\lambda\xi & 0 \\ 0 & \phi \end{bmatrix},$$

So the quality characteristic will be stable if and only if $\left|1-\lambda\xi\right| < 1$ and $\left|\phi\right| < 1$. From (A1)-(A4), assuming null initial condition, we get

$$\lim_{k\to\infty} E[y_k] = T \,. \tag{24}$$

Similarly, from (A1)-(A4) it can be shown after some algebraic operation that

$$\lim_{k\to\infty} \mathrm{var}[y_k] = \frac{2\{1+\theta[\theta-2\phi+\lambda\xi(1+\phi)]\}}{(2-\lambda\xi)(1+\phi)[1-(1-\lambda\xi)\phi]}\sigma_\varepsilon^2 \,. \tag{25}$$

For ARIMA(1,1,1) disturbance

For the ARIMA(1,1,1) disturbance, one uses the system state equation (6)

$$\begin{bmatrix} \delta_{k+1} \\ \phi\delta_k \end{bmatrix} = \begin{bmatrix} 1+\phi & 1 \\ -\phi & 0 \end{bmatrix} \begin{bmatrix} \delta_k \\ \phi\delta_{k-1} \end{bmatrix} + \begin{bmatrix} 1 \\ -\theta \end{bmatrix} \varepsilon_{k+1} \,, \tag{A5}$$

and the output equation,

$$y_k = \alpha + \beta u_k + \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \delta_k \\ \phi\delta_{k-1} \end{bmatrix} . \tag{A6}$$

Hence, the state-space equation, (A3), becomes

$$\mathbf{x}_{k+1} = \mathbf{A}\mathbf{x}_k + \mathbf{w}_k$$

$$y_k = \mathbf{C}\mathbf{x}_k + R \tag{A7}$$

where

$$\mathbf{x}_k = \begin{bmatrix} \hat{d}_{k-1} \\ \delta_k \\ \phi\delta_{k-1} \end{bmatrix}, \quad \mathbf{A} = \begin{bmatrix} 1-\lambda\xi & \lambda & 0 \\ 0 & 1+\phi & 1 \\ 0 & -\phi & 0 \end{bmatrix},$$

$$\mathbf{w}_k = \begin{bmatrix} \lambda\left(\alpha-\xi\alpha+\xi T-T\right) \\ \varepsilon_{k+1} \\ -\theta\varepsilon_{k+1} \end{bmatrix},$$

$$\mathbf{C} = \begin{bmatrix} -\xi & 1 & 0 \end{bmatrix},$$

and $R = \alpha + \xi T$. From equations (A5)-(A7) and (7), assuming null condition, one obtains equations (26) and (27).

Appendix B

Derivation of asymptotic mean and variance of the KF controller for DT, RWD, IMA(1,1), ARMA(1,1) and ARIMA(1,1,1) disturbances.

The derivation procedure is almost the same for the cases of DT, RWD, IMA(1,1) ARMA(1,1) and ARMA(1,1,1) disturbances; thus, we only demonstrate the ARMA(1,1) disturbance as an example. Combining equations (9), (10), (28), (29) and (5), we can rewrite (A3) as

$$\mathbf{x}_{k+1} = \mathbf{A}\mathbf{x}_k + \mathbf{w}_k$$

$$y_k = \mathbf{C}\mathbf{x}_k + R_k \tag{B1}$$

where

$$\mathbf{X}_k = \begin{bmatrix} \hat{\delta}_k \\ \phi\hat{\delta}_{k-1} \\ \delta_k \\ \phi\delta_{k-1} \end{bmatrix}, \quad \mathbf{A} = \begin{bmatrix} 1-\xi K_1 & 0 & K_1 & 0 \\ -\xi K_2 & 1 & K_2 & 0 \\ 0 & 0 & \phi & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\mathbf{C} = \begin{bmatrix} -\xi & 0 & 1 & 0 \end{bmatrix},$$

$$\mathbf{w}_k = \begin{bmatrix} K_1\ \left(\xi T - T - \xi\alpha + \alpha + v_k\right) \\ K_2\ \left(\xi T - T - \xi\alpha + \alpha + v_k\right) \\ \varepsilon_{k+1} \\ -\theta\varepsilon_{k+1} \end{bmatrix},$$

and $R_k = \xi T + \alpha - \xi\alpha + v_k$. The solution of $\mathbf{x}_k$ is given by

$$\mathbf{x}_k = \mathbf{A}^k \mathbf{x}_0 + \sum_{i=1}^{k-1} \mathbf{A}^{k-i-1}\mathbf{w}_i \tag{B2}$$

Thus for distinct eigenvalues, $\mathbf{A}^k$ can be computed by $\mathbf{A}^k = \mathbf{P}\mathbf{\Gamma}^k\mathbf{P}^{-1}$

$$\mathbf{\Gamma} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \phi & 0 \\ 0 & 0 & 0 & 1-\xi K_1 \end{bmatrix}.$$

If $\mathbf{A}$ has multiple eigenvalues, the $\mathbf{A}^k$ should be computed from $\mathbf{M}\mathbf{J}^k\mathbf{M}^{-1}$ where $\mathbf{J}$ is a matrix in Jordan canonical form and $\mathbf{M}$ is a matrix of eigenvectors of $\mathbf{A}$. From equations (B1)-(B2), assuming null initial condition and taking expectation with output, we can obtain equations (36) and (37).

Appendix C

The optimum gain of the Kalman filter with measurement noise for five disturbances can be obtained by taking the derivative of AMSD with respect to K_1. If the disturbance is DT, the AMSD of output is

$$\mathrm{AMSD}_{DT} = \frac{2\sigma_\varepsilon^2}{2-K_1\xi} + \frac{\Delta^2}{K_1^2\xi^2} + \frac{2\sigma_v^2}{2-K_1\xi} \,. \tag{C1}$$

The optimum gain can be obtained by taking the derivative and setting to zero.

$$\left(\sigma_\varepsilon^2+\sigma_v^2\right)K_1^3\xi^3-\left(K_1\xi-2\right)^2\Delta^2=0 \tag{C2}$$

The solution of equation (C2) should satisfy the stability condition $\left|1-K_1\xi\right|<1$. Note that the roots of equation (C2) have to be real. In the RWD disturbance case, the AMSD is

$$\begin{aligned}&\text{AMSD}_{RWD}\\&=\frac{\sigma_\varepsilon^2}{K_1\xi(2-K_1\xi)}+\frac{\Delta^2}{K_1^2\xi^2}+\frac{2\sigma_v^2}{2-K_1\xi}.\end{aligned} \tag{C3}$$

Taking the derivative and setting it to zero, we get:

$$K_1^3\xi^3\sigma_v^2+K_1\xi\left(-1+K_1\xi\right)\sigma_\varepsilon^2-\delta^2\left(K_1\xi-2\right)^2=0 \tag{C4}$$

The optimum gain of the Kalman filter under RWD disturbance can be obtained by solving equation (C4). The root of equation (C4) should obey the stability condition as in DT case. If the disturbances are IMA(1,1) and ARMA(1,1), we have the following results:

$$\text{AMSD}_{IMA}=\frac{[1-2(1-K_1\xi)\theta+\theta^2]}{K_1\xi(2-K_1\xi)}\sigma_\varepsilon^2+\frac{2\sigma_v^2}{2-K_1\xi} \tag{C5}$$

The optimum gain for IMA(1,1) disturbance is:

$$\begin{aligned}&K_{1,IMA}\\&=\frac{-(\theta-1)^2\sigma_\varepsilon^2\pm(\theta-1)\sigma_\varepsilon\sqrt{4\sigma_v^2+(1+\theta)^2\sigma_\varepsilon^2}}{2\xi\left(\sigma_v^2+\theta\sigma_\varepsilon^2\right)}\end{aligned} \tag{C6}$$

and

$$\begin{aligned}&\text{AMSD}_{ARMA}\\&=\frac{2\{1+\theta[\theta-2\phi+K_1\xi(1+\phi)]\}}{(2-K_1\xi)(1+\phi)[1-(1-K_1\xi)\phi]}\sigma_\varepsilon^2+\frac{2\sigma_v^2}{2-K_1\xi}\end{aligned} \tag{C7}$$

The optimum gain for ARMA(1,1) disturbance is:

$$\begin{aligned}K_{1,ARMA}&=\frac{\phi\left[\left(\phi^2-1\right)\sigma_v^2-\left(1+\theta^2-2\theta\phi\right)\sigma_\varepsilon^2\right]}{\xi\phi\left(1+\phi\right)\left(\phi\sigma_v^2+\theta\sigma_\varepsilon^2\right)}\\&\pm\frac{\sigma_\varepsilon\sqrt{\phi\left(\theta-\phi\right)\left(\theta\phi-1\right)\left[\left(1+\phi\right)^2\sigma_v^2+\left(1+\theta\right)^2\sigma_\varepsilon^2\right]}}{\xi\phi\left(1+\phi\right)\left(\phi\sigma_v^2+\theta\sigma_\varepsilon^2\right)}\end{aligned} \tag{C8}$$

By the same procedure as in the DT and RWD disturbance cases, the AMSD for ARIMA(1,1,1) disturbance can be obtained as follows.

$$\begin{aligned}&\text{AMSD}_{ARIMA}\\&=\frac{(1+\phi-K_1\xi\phi)(\theta^2+1)-2\theta(1+\phi-K_1\xi)}{K_1\xi(2-K_1\xi)(1-\phi^2)[1-(1-K_1\xi)\phi]}\sigma_\varepsilon^2\\&+\frac{2\sigma_v^2}{2-K_1\xi}\end{aligned} \tag{C9}$$

The optimum gain for ARIMA(1,1,1) can be obtained by solving equation (C10)

$$\begin{aligned}&K_1^2\xi^2\left[1+\left(K_1\xi-1\right)\phi\right]^2\left(\phi^2-1\right)\sigma_v^2\\&+\Big\{\left(1+\theta^2\right)\Big[1-\phi^2+K_1^3\xi^3\phi^2-K_1^2\xi^2\phi\left(1+3\phi\right)\\&+K_1\xi\left(-1+2\phi+3\phi^2\right)\Big]-\theta\Big[2+2K_1^3\xi^3\phi-2\phi^2\\&+K_1^2\xi^2\left(1-6\phi-3\phi^2\right)+2K_1\xi\left(-1+2\phi+3\phi^2\right)\Big]\Big\}\sigma_\varepsilon^2\\&=0\end{aligned} \tag{C10}$$

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